Chapter 10

Flow Measurements

Material from Theory and Design for Mechanical Measurements;
Figliola, Third Edition

Flow Rate

- Flow rate can be expressed in terms of volume flow rate (volume/time) or mass flow rate (mass/time).
- Flow rate through pipe or duct can be defined using concept of control volume, where the amount of flow through volume in a given period of time is the flow rate.

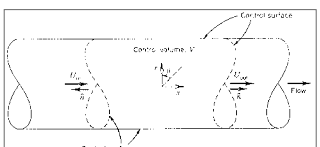

Figure 10.1 Control volume concept as applied to flow through a pipe.

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Flow Rate

- The velocity of the flow is described at any point by: $\mathbf{U}(\mathbf{x}, \mathbf{r}, \vartheta) = \mathbf{u}\hat{\mathbf{e}}_x + \mathbf{v}\hat{\mathbf{e}}_r + \mathbf{w}\hat{\mathbf{e}}_\vartheta$
  - u, v, and w are scalar velocity magnitudes and $\hat{\mathbf{e}}_x, \hat{\mathbf{e}}_r, \hat{\mathbf{e}}_\vartheta$ are unit vectors.
- By conservation of mass, the amount of fluid density ρ that passes through control volume (CV) and accumulates within CV must be equal to 0.

Flow Rate

- For steady flows, $\overset{o}{\mathbf{m}}_{in} = \overset{o}{\mathbf{m}}_{out}$
  - where $\overset{o}{\mathbf{m}} = \int\int_A \rho \mathbf{u} * \hat{\mathbf{n}} \mathbf{dA}$
  - $\overset{o}{\mathbf{m}}$ is mass flow rate through area A
- For isometric flows, consider liquids as incompressible.
- $Q_{in} = Q_{out}$ where $\mathbf{Q} = \int\int_A \mathbf{u} * \hat{\mathbf{n}} \mathbf{dA}$
  - Where Q is the volumetric flow rate

Flow Rate

- In pipe flow, if we know $u(r,\vartheta)$ at position x, in steady, incompressible flow, we can estimate the average velocity and, thus, average flow.
  - $\mathbf{Q} = \int_0^{r_1}\int_0^{2\pi} \mathbf{u}(\mathbf{r},\vartheta)\mathbf{r}\mathbf{d}\vartheta\mathbf{dr}$
  - r_1 is the pipe radius.

Flow Rate

- If the average velocity is known, $Q = \overline{\mathbf{U}}A$.
- A method that will measure steady flow rates must be sensitive to either average mass flux $\overline{\rho \mathbf{U}}$ or average velocity $\overline{\mathbf{U}}$.

Flow Rate

- The flow in a pipe is either laminar, turbulent, or transitional.
- Using Reynolds number, $R_e = \overline{U} d_1 / v = \frac{4Q}{\pi d_1 v}$
  - Turbulent flow: $R_e > 4000$ v = kinematic viscosity
  - Laminar flow: $R_e < 2000$
  - d_1 = diameter for circular pipe or hydraulic diameter ($4r_h$) for non-circular cross- sections.
  - r_h = wetted area/ wetted parameter
  - Note: Stay away from transitions in flow

Volume Flow Rate

- The direct implementation of equations for estimating the volume flow rate through a duct requires measurement of the velocity at points along several cross-sections of a flow control surface.
- This procedure is most often used for the one-time verification or calibration of system flow rates.

Volume Flow Rate

- In using this technique in circular pipes, a number of discrete measuring positions are chosen along "*m*" flow cross-sections (radii) spaces at $360^o/m$ apart.
- A velocity probe is traversed along each flow cross-section with readings taken at each measurement position.

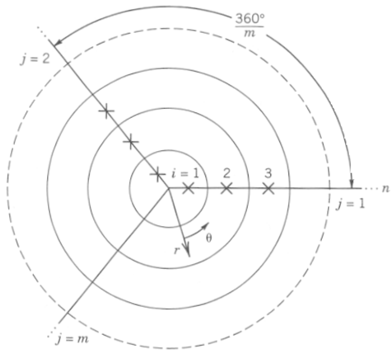

Figure 10.2 Location of *n* measurements along *m* radial lines in a pipe.

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Obstruction Flow Meters

- Three types of meters that use obstruction:
  1. Orifice plate
  2. Venturi
  3. Flow Nozzle

Obstruction Flow Meters

- Obstruction flow meters rely on the Bernoulli effect. When a restriction in flow occurs, there is an increase in velocity (conservation of mass). When velocity goes up, pressure goes down. Therefore, there is a measurable pressure drop across the orifice/obstruction.
- $\Delta p = p_1 - p_2$ $\quad\quad Q \approx (p_1 - p_2)^n$
  - n = 1 (laminar) n = ½ (turbulent)

Obstruction Meters

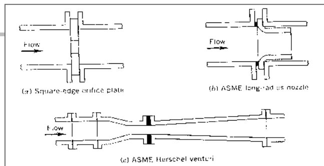

Figure 10.3 Flow area profiles of common obstruction meters.

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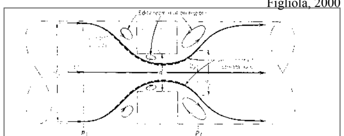


Figure 10.4 Control volume concept as applied between two streamlines for flow through an obstruction meter.

- Placed in line with pipe
- Due to vena contracta effect
  - Due to inability of flow to instantaneously adjust to change in cross-section.
- Assume:
  1. No external heat energy added
  2. No shaft work done
  3. Steady flow
  4. One-dim flow

Energy Equation

- $p_1/\gamma + \bar{U}_1^2/2g = p_2/\gamma + \bar{U}_2^2/2g + h_{L1-2}$
- From conservation of mass, $\bar{u}_1 = \bar{u}_2 A_2/A_1$
- For incompressible flow,

$$Q_I = \bar{U}_2 A_2 = (A_2/[1-(A_2/A_1)^2]^{1/2})\sqrt{[2(p_1-p_2)/\rho] + 2gh_{L1-2}}$$

- This equation can be simplified using coefficients

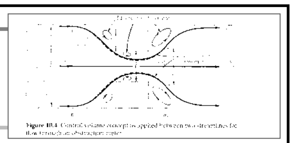
Obstruction Meters

- When the flow area changes abruptly, the effective flow area immediately downstream of the alteration will not necessarily be the same as the pipe flow area.
- This effect is brought about by an inability of a fluid to expand immediately upon encountering an expansion as a result of the inertia of each fluid particle.
- This forms a central core flow bounded by regions of slower moving recirculating eddies.

Obstruction Meters

- As a consequence, the pressure sensed with pipe wall taps located within the vena contracta region will correspond to the higher moving velocity within the vena contracta of unknown flow area, A_2.
- The unknown vena contracta area will be accounted for by introducing a contraction coefficient, C_c.

Energy Equation



- $C_c = A_2/A_0$ contraction coefficient →

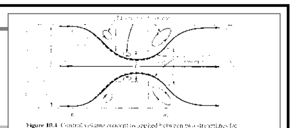
$$Q_I = C_c A_0 / [1 - (C_c A_0 / A_1)^2]^{1/2} \sqrt{2(\Delta P)/\rho + 2gh_{21-2}}$$

- C_f = frictional head loss →

$$Q_I = C_f C_c A_0 / [1 - (C_c A_0 / A_1)^2]^{1/2} \sqrt{2(\Delta P)/\rho}$$

- C = discharge coefficient combines C_c and C_f

Energy Equation



- $Q_I = CEA\sqrt{2\Delta p/\rho}$ where E is the velocity of the approach factor.
- $E = 1/[1-(A_o/A_1)^2]^{1/2} = 1/(1-\beta^4)^{1/2}$ where $\beta = d_0/d_1$
- CE is often represented as flow coefficient K_o
  - The value of C varies depending on the flow, Reynolds number and the β ratio.

Energy Equation

- Because the magnitude of vena contracta and head loss vary along the length of meter the location of the pressure taps is critical.
- The flow behavior of orifice plate, venturi, and flow nozzle meters have been studied extensively and are therefore used without calibration through use of standard flow factor tables and hand books.

Orifice Meter

- An orifice meter consists of a circular plate, containing a hole (orifice), which is inserted into a pipe such that the orifice is concentric with the pipe inside diameter.
- Several variations in the orifice design exist, but the square-edged orifice is common.
- Installation is simplified by housing the orifice plate between two pipe flanges.

Orifice Meter

- With this technique, an orifice plate is interchangeable with others of different β values
- The simplicity of the installation and orifice design allows for a range of β values to be maintained on hand at modest expense.

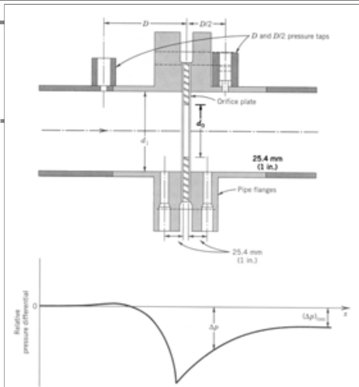


Figure 10.5 Square-edged orifice meter installed in a pipeline with optional one pipe and one-half pipe diameters, and flange pressure taps shown. Relative flow pressure drop along the pipe axis is shown.

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Venturi Meter

- A venturi meter consists of a smooth converging contraction to a narrow throat followed by a shallow diverging section.
- The standard venturi can utilize either a 15^o or 7^o divergent section.
- The meter is installed between two flanges intended for this purpose.
- Pressure is sensed between a location upstream of the throat and a location at the throat such that equations are used with values for both A and β based on the throat diameter.

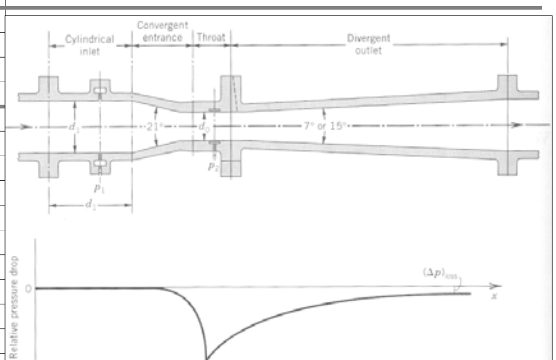

Figure 10.9 Herschel venturi meter with the associated flow pressure drop along its axis.

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Flow Nozzles

- A flow nozzle consists of a gradual contraction to a narrow throat.
- It needs less installation space than a venturi meter and has ~80% of the initial cost.
- A common form for the nozzle is the ASME long radius nozzle, in which the nozzle contraction is that of the quadrant of an ellipse with major axis aligned with the flow axis.

Flow Nozzles

- The nozzle is typically installed in line, but can also be used at the inlet to and the outlet from a plenum or reservoir or at the outlet of a pipe.
- Pressure taps are usually located at one pipe diameter upstream of the nozzle inlet and at the nozzle throat by using either wall or throat taps.
- The flow rate is determined from equations with values for A and β based on the throat diameter.

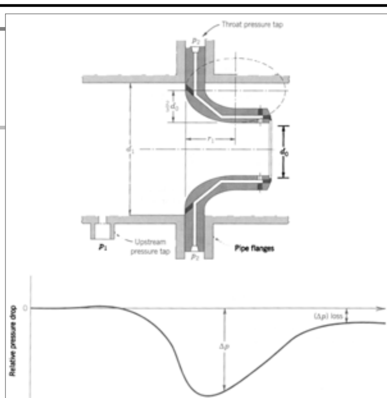

Figure 10.10 ASME long-radius nozzle with the associated flow pressure drop along its axis.

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Sonic Nozzles

- "Sonic nozzles are used to meter and to control the flow rate of compressible gases. They may take the form of any of the previously described obstruction meters. If the gas flow rate through and obstruction meter becomes sufficiently high, the sonic condition will be achieved at the meter throat. At the sonic condition, the gas velocity will equal the acoustic wave speed (speed of sound) of the gas. At that point, the throat is considered to be choked and the mass flow rate through the throat will be at a maximum for the given inlet conditions regardless of any further increase in pressure drop across the meter."

Laminar Flow Elements

• Laminar flow elements force the fluid flow to be laminar and then use expressions for laminar flow to interpret the Pressure drop.

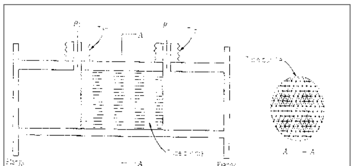

Figure 10.14 Laminar flow element flow meter.

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Laminar Flow

- $p_1 / \gamma + \bar{U}_1^2 / 2g = p_2 / \gamma + \bar{U}_2^2 / 2g + h_{L_{1-2}}$
- Darcy-Weisback for common flow:
  - $h_{L_{1-2}} = f(L / d_1)(U_1^2 / 2g)$ when $f = 64 / (R_e d_1)$
  - f is the friction factor
- $Q = \bar{U}_1(\pi d_1^2 / 4) = \bar{U}_2(\pi d_2^2 / 4)$ for $d_1=d_2$ → $\bar{U}_1 = \bar{U}_2$
  - Thus $p1 - p2 = \gamma h_{L_{1-2}}$ using $\bar{U} = 4Q / \pi d_1^2$

$$Q = (\pi d_1^4 / 128\mu)(p_1 - p_2 / L)$$

L = length of tube section

Advantages/Disadvantages

- Advantages:
  1. High sensitivity at low flow rates
  2. Can measure flow in either direction
  3. Wide flow range
  4. Indicated average flow in pulsing flow condition
- Disadvantages:
  1. Susceptible to plugging, only use in clean fluids
  2. All measured pressure drop, remains a system drop

Insertion Volume Flow Meters

- The largest group of meters based on a phenomenon that is actually sensitive to the average velocity across a control surface of known area.
- Several of these designs are included in the discussion that follows.

Electromagnetic Flow Meters

- The operating principle of an electromagnetic flow meter is based on the fundamental principle that an emf of electric potential, E, is induced in a conductor of length, L, which moves with a velocity, U, through a magnetic field of magnetic flux, B.

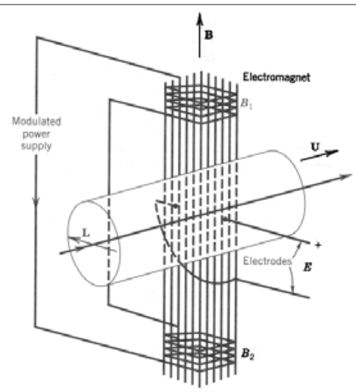

Figure 10.15 Electromagnetic principle as applied to a working flow meter.

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Electromagnetic Flow Meters

- The electromagnetic flow meter comes commercially as a packaged flow device, which is installed directly in line and connected to an external electronic output unit.
- Units are available using either permanent magnets, called dc units, or variable flux strength electromagnets, called ac units.

Electromagnetic Flow Meters

- The electromagnetic flow meter has a low pressure loss associated due to its open tube, no obstruction design, and it is suitable for installations that can tolerate only a small pressure drop.
- This absence of internal parts is very attractive for metering corrosion and "dirty" fluids.
- The operating principle is independent of fluid density and viscosity, responding only to average velocity and there is no difficulty with measurements in either laminar or turbulent flows, provided that the velocity profile is reasonably symmetrical.

Vortex Shedding Meters

- Vortex shedding is a natural phenomenon where alternating vortices are shed in the wake of a body at a frequency that depends on the flow velocity past the body.
- The vortices formed on opposite sides of the body are carried downstream in the body's wake forming a "vortex street," each vortex having an opposite sign of rotation.
- The vortex shedding phenomenon is used to sense average velocity in pipe flows in a vortex flow meter.

Vortex Shedding Meters

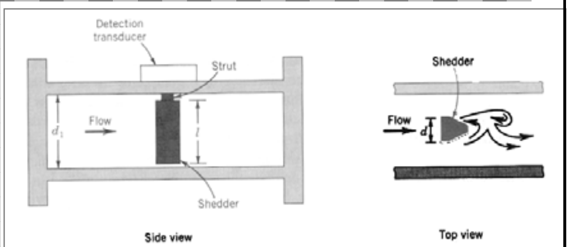

Figure 10.17 Vortex shedding flow meter. Different shedder shapes are available.

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Vortex Shedding Meters

- The lower flow rate limit on vortex meters appears to be at Reynolds numbers near 10,000.
- This can be a problem in metering high-viscosity pipe flows.
- Density variations affect the strength of the shed vortex and this places a lower limit on fluid density which is based on the sensitivity of the vortex shedding detection equipment.
- The meter has no moving parts and relatively low pressure compared to obstruction meters.

Turbine Meters

- The exchange of momentum between the flow and a rotor turns the rotor at a rotational speed that is proportional to the flow rate.
- A rotor is encased in a bored housing that is inserted in the flow stream to be metered.
- Turbine meters have a low-pressure drop and are very accurate.

Turbine Meters

- They are exceptionally repeatable, but are restricted to clean fluids because of possible fouling of their rotating parts.
- They are sensitive to temperature changes which affect fluid viscosity.

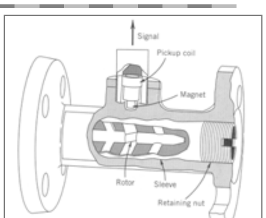

Figure 10.19 Cutaway view of a turbine flow meter. (Courtesy of Schlumberger Industries, Measurements Division, Greenwood, SC.)

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