Chapter 11

SERVO VALVES (contd.)

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Servo Analysis

- The concept of gain has preciously been defined

  G = Output / Input

- Feedback principle : We sense the output, the error between input and feedback loop drives the system to the desired zero error condition, thus feedback helps control the system.

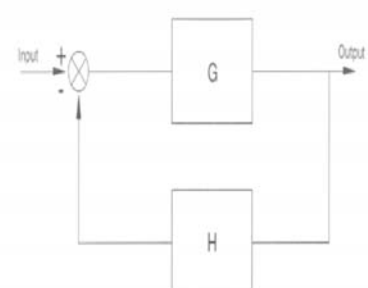

FIGURE 11.34
Block diagram of a system with feedback.

Servo Analysis

- Feedback signal will be opposite in sign to the input signal.
- When correcting for drift in the output, we must move it back toward the set point. This correction is called negative feedback.
- Closed loop hydraulic systems are also called servo systems.

Servo Analysis

- Feedback signal is typically a scaled DC voltage, which is proportional to the output signal.
- If the feedback signal is an AC sine wave it must be shifted in phase by 180^{o} from the input.
- When the amplitudes are equal, the two signals cancel each other, and the resulting error is zero.

Servo Analysis

- Block diagram of closed-loop system for servo cylinder.
- The servo valve transfer function is the flow transfer function, not the pressure transfer function.
- Input to the cylinder is a flow, in^3/s, and the output is a linear velocity, in/s.

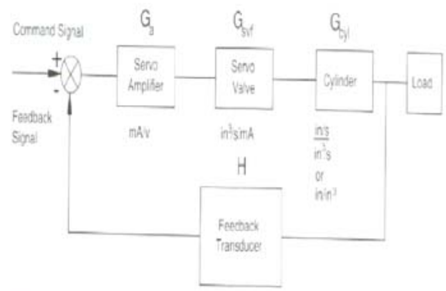

FIGURE 11.35
Block diagram of a servo cylinder closed-loop system.

Servo Analysis

- Transfer function is given by ,

$$G_{cyl} = \frac{\text{output}}{\text{input}} = \frac{in/s}{in^3/s} = \frac{in}{in^3} = \frac{1}{in^2} = \frac{1}{A}$$

where A = cylinder area (in^2).

- A typical feedback transducer is the potentiometer. It’s transfer function is V/in. A linear velocity (in/s) drives the potentiometer to produce the feedback signal (V), not V/s.

Servo Analysis

- **Open-Loop Gain**
- defined by

$$GH = G_a \times G_{svf} \times G_{cyl} \times H$$
$$= (mA/V) \times ((\text{in}^3/\text{s})/\text{mA}) \times (\text{in}/\text{in}^3) \times (\text{V}/\text{in})$$
$$= 1/\text{s} \qquad \text{.............(Eq 11.23)}$$

Servo Analysis

- Open loop gain is also referred to as the velocity constant.

$$k_v = GH = G_a \times G_{svf} \times G_{cyl} \times H \qquad \text{.........(Eq 11.24)}$$

Servo Analysis

- **Natural Frequency**
- The analysis required to obtain the natural frequency of a servo cylinder connected to a load will be calculated.
- Model for this system shown in Fig 11.36

Servo Analysis

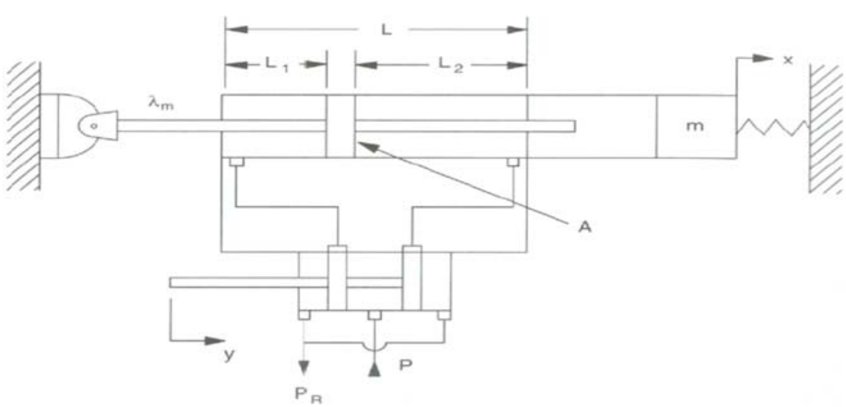

FIGURE 11.36
Diagram of servo cylinder moving a load [model of hydromechanical servo (Fig. 11.27)].

Servo Analysis

- The mass is held in position with a spring which suggests that the load has mass and elasticity.
- Stiffness describes the force required to produce a unit deflection.
- Compliance is the reciprocal of stiffness, thus the units are deflection per unit force, or in/lb_f

Servo Analysis

- The system (Fig 11.36) can be visualized as a mass held in position by a column of fluid.
- If compliance of this fluid is λ_o, and the compliance of rod is λ_m, then the total cylinder compliance is

$$\lambda = \lambda_o + \lambda_m$$

- Generally, $\lambda_m << \lambda_o$**,** so the compliance of the oil is used as the cylinder compliance with negligible error.

Servo Analysis

- Hydraulic oil compresses when pressure is applied.
- Relationship termed bulk modulus is defined by

$$\beta = V \frac{\Delta P}{\Delta V}$$

where β = bulk modulus (psi)

V = original volume before pressure is applied (in^3)

ΔP= applied pressure (psi)

ΔV= change in volume (in^3)

Servo Analysis

- Suppose force F is applied to cylinder with effective piston area A. The column of fluid has length L_1.
- Pressure resulting from application of force is

$$\Delta P = F / A \quad \text{or} \quad F = \Delta P A$$

Servo Analysis

- Displacement is ΔL. Using definition of compliance,

$$\lambda = \Delta L / F$$
$$= \Delta L / \Delta PA$$
$$= \Delta LA / \Delta PA^2$$

Servo Analysis

- Change in volume is given by

$$\Delta V = \Delta LA$$

Therefore,

$$\lambda = \Delta V / \Delta PA^2 \quad \text{..........(Eq 11.30)}$$

Servo Analysis

- From the definition of bulk modulus,

$$\Delta V / \Delta P = V / \beta \quad ....(Eq\ 11.31)$$

Substituting Eq(11.31) into Eq.(11.30)

$$\lambda = V / \beta A^2 .... \; ....(Eq\ 11.32)$$

Servo Analysis

- The original volume is V= AL_1, and substituting in Eq(11.32),

$$\lambda = L_1 / \beta A$$

- Compliance of oil columns at both ends of cylinder is given by,

$$\lambda_{01} = L_1 / \beta A, \quad \lambda_{02} = L_2 / \beta A$$

Servo Analysis

- The two columns of fluid are in series.
- The cylinder body with rigidly attached valve body and mass shown in Fig 11.36 can be represented by the mass m shown in Fig 11.37
- The equivalent stiffness is

$$k = k_1 + k_2$$

Servo Analysis

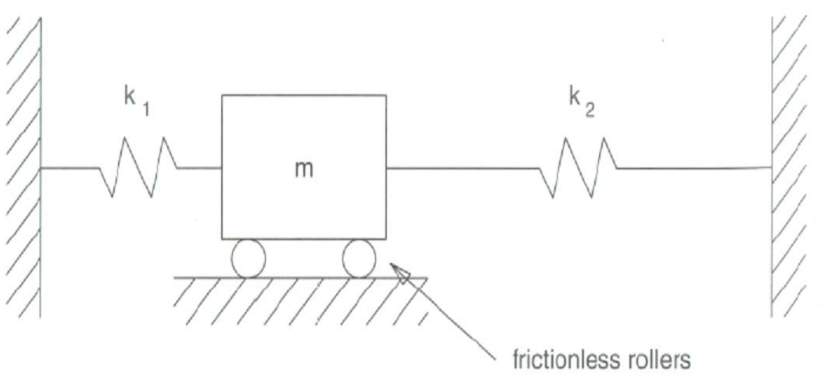

FIGURE 11.37
Model of servo cylinder showing actuator mass supported by columns of fluid on both sides.

Servo Analysis

- Remembering that compliance is the reciprocal of stiffness,

$$1/\lambda_0 = (1/\lambda_{01}) + (1/\lambda_{02})$$

Solving for λ_0, the equivalent compliance

$$\lambda_0 = (\lambda_{01}\lambda_{02})/(\lambda_{01} + \lambda_{02}) \text{ ....(Eq11.35)}$$

Servo Analysis

- Substituting for λ_{01} and λ_{02}

$$\lambda_0 = L_1 L_2 \; / \; \beta A \, [L_1 + L_2] \text{........Fi g 11.35}$$

- When $L_1 = L_2 = L/2$, λ_0 is maximum.

$$\lambda_{0(\max)} = L / 4\beta A \text{ ....Fig 11.37}$$

- Eq11.37 is typically used for cylinder compliance

Servo Analysis

- When the valve is not mounted directly on the cylinder, the oil in the connecting line affect performance.
- The oil in the lines between the valve and cylinder must be considered.

Servo Analysis

- Let the total oil volume in both lines is V_{line}.
- If volume change (swelling) in the lines is neglected, giving a constant V_{line}, cylinder movement due to compressibility of oil in V_{line} can be calculated as follows.

Servo Analysis

- Visualize the cylinder being extended on both ends to accommodate line volume on each end.

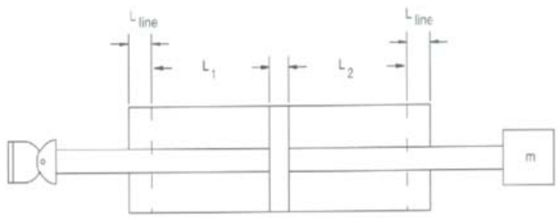

FIGURE 11.38
Diagram of servo valve cylinder showing increase in fluid column length to account for fluid in lines.

Servo Analysis

- Length of extension is

$$L_{line} = (V_{line} / 2) / A$$

- Effective length of column of fluid on both ends is

$$L_{eff1} = L_1 + L_{line}$$

$$L_{eff2} = L_2 + L_{line}$$

Servo Analysis

- **Compliance** is

$$\lambda_{01} = (L_1 + L_{line}) / \beta A$$

$$\lambda_{02} = (L_1 + L_{line}) / \beta A \quad \text{.......... Eq 11.39}$$

Servo Analysis

- Substituting in Eq11.35, we get

$$\lambda_o = \frac{[\frac{L_1 + L_{line}}{\beta A}][\frac{L_2 + L_{line}}{\beta A}]}{[\frac{L_1 + L_{line}}{\beta A}] + [\frac{L_2 + L_{line}}{\beta A}]}$$

Servo Analysis

- Substituting $L_1 = L_2 = L/2$ and simplifying, we get

$$\lambda_{o(\max)} = [\frac{L/2 + L_{line}}{2\beta A}]$$

$$= [\frac{L}{4\beta A} + \frac{L_{line}}{2\beta A}] \quad \ldots\ldots\ldots\ldots\ldots\ldots (Eq.11.41)$$

Servo Analysis

- Substituting Eq(11.38) into Eq(11.41),

$$\lambda_o = [\frac{L}{4\beta A} + \frac{(V_{line})}{4\beta A}]$$

$$= \frac{(LA + V_{line})}{4\beta A^2} \ldots\ldots\ldots\ldots\ldots\ldots (Eq.11.42)$$

Servo Analysis

- The term (LA +V_{line}) is the total volume of fluid in the cylinder and the lines.
- Natural frequency is defined by

$$\omega = \sqrt{\frac{1}{\lambda_o m}} = \frac{1}{\sqrt{\lambda_o m}}$$

where ω = natural frequency (rad/s)
λ_o = compliance (in/lb_f)
m = mass ($lb_f.s^2$/in.)

Servo Analysis

- Substituting from Eq.(11.42),

$$\omega = \sqrt{\frac{4\beta A^2}{Vm}}$$

Where ω = natural frequency (rad/s)
m = load mass ($lb_f.s^2$/in.)
A = cylinder area(in^2)
β = bulk modulus (lb_f/in^2)
V = total volume of fluid in cylinder and lines (in^3)

Servo Analysis

- Servo systems should be designed for the velocity constant fall in the range between 1/2 and 1/3 of the natural frequency.

- Velocity constant, $k_v > \omega/2$ will result in unstable system.

Servo Analysis

- Two types of instability can be observed.
  - A step input will produce a damped vibration that settles out after a few oscillations.
  - Second type of instability, corresponding to a higher k_v, is a continuous oscillation.
- For more conservative k_v, select an open-loop gain such that $k_v = \omega/3$.

Servo Analysis

- Generally, a set of components is selected, their transfer functions calculated, and then the amplifier gain is selected to ensure that k_v does not exceed ω/3.

Servo Analysis

- **Error Terms**
  - Servo systems can achieve excellent accuracy, but position (or velocity) control is never exact.
- **Position error**
  - Position error is defined by

$$Position\ error = \frac{\sum component\ deadbands}{Amplifier\ gain \times Feedback\ gain} \quad \text{..........(Eq.11.46)}$$

Servo Analysis

- Most significant component deadband is the servo valve threshold.
- Other component deadbands are significantly smaller and can often ignored.
- If the servo valve threshold is 3mA, the amplifier gain is 190 mA/V, and the feedback potentiometer transfer function is 6 V/in.

Servo Analysis

- Position error is

$$E_p = \frac{3mA}{190\,mA/V \times 6\,V/in} = 0.0026\,in$$

- We cannot simply increase amplifier gain to achieve smaller position error. K_v must be kept below ω/3. Increasing G_a would increases K_v and can drive the system unstable.

Servo Analysis

- Following factors influence position error of a servo cylinder.

- Load mass: An increase in mass gives an increase in position error. Larger mass reduces natural frequency.

$$\omega = \sqrt{\frac{4 \beta A^2}{Vm}}$$

Servo Analysis

- Reducing ω reduces K_v . Solving for G_a,

$$G_a = \frac{k_v}{G_{sv} \times G_{cyl} \times H}$$

- A smaller K_v means a smaller G_a, thus

$$E_p = \frac{deadband}{G_a \times H}$$

Servo Analysis

- An increase in cylinder stroke gives an increase in position error, since a larger cylinder volume reduces the natural frequency, which reduces K_v ($K_v \leq \omega/3$), which reduces G_a , which increase E_p.

- An increase in cylinder bore often reduces the position error,since the natural frequency will increase, K_v can be increased, which allows an increase in G_a , which can reduce E_p.

Servo Analysis

- **Tracking Error**

- Tracking error is the error between the command and feedback voltages while the command voltage is changing.

- Consider a system where one cylinder is “slaved” to another cylinder. (Fig 11.39)

- The ratio adjust sets the ratio of movement of the two cylinders.

Servo Analysis

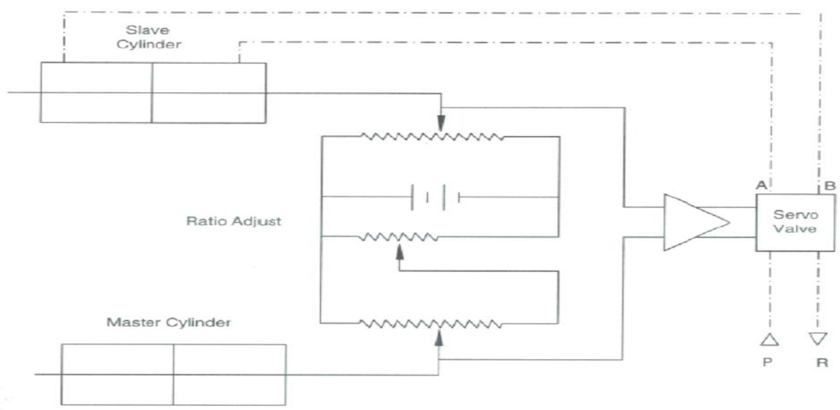

FIGURE 11.39
System for slaving one cylinder to another.

Servo Analysis

- For the slave cylinder to move only 50% of the master cylinder movement, the ratio adjust is set to 50%.
- When the two cylinders are moving there will be a difference in their position.
- Unless master cylinder leads the slave cylinder by some amount, there is no error signal, and the servo valve will not open.

Servo Analysis

- Tracking error is defined as

$$E_t = \frac{\text{cylinder velocity}}{G \times H}$$

$$= \frac{\text{cylinder velocity}}{G_a \times G_{svf} \times G_{cyl} \times H}$$

Servo Analysis

- For the servo cylinder shown in Fig 11.36, the input is the spool displacement (y), and output is load displacement (x).
- Transfer function for the servo cylinder combination is:

$$G = \frac{\text{output}}{\text{input}} = \frac{x}{y}$$

Servo Analysis

- Load displacement is given by

$$x = \frac{1}{A}\int Qdt$$

Where x = displacement (in)
A = cylinder area (in^2)
Q = flow rate (in^3/s)
t = time (s)

Servo Analysis

- Full displacement is not achieved due to fluid compliance.
- Including compliance, the actual displacement is

$$x = \frac{1}{A}\int Qdt - F\lambda \qquad \text{........} (11.56)$$

Where F = force (lb_f)
λ = compliance (in/ lb_f)

Servo Analysis

- To transform into Laplace domain, the integral is replaced with 1/s; therefore (Eq.11.56) becomes

$$x = \frac{1}{As}Q - F\lambda \text{ ....(11.57)}$$

- Assuming that supply pressure is a constant, which is valid when a good quality relief valve is used in the supply circuit, flow is a function of two variables, spool displacement and load pressure.

$$Q = f(y, P_L)$$

Servo Analysis

- If the servo cylinder is operating at steady-state, and if the changes in y and P_L about this point are small, flow can be approximated by

$$Q = \frac{\Delta Q}{\Delta y}y + \frac{\Delta Q}{\Delta P_L}P_L \text{..................(11.58)}$$

- Substituting Eq11.58 into Eq11.57, we get,

$$x = \frac{1}{As}[\frac{\Delta Q}{\Delta y}y + \frac{\Delta Q}{\Delta P_L}P_L] - \lambda F$$

Servo Analysis

Pressure factor

- The change in flow through the valve per unit change in load pressure. It is defined by

$$P_f = \frac{\Delta Q}{\Delta P_L}$$

Servo Analysis

- When load pressure is close to supply pressure, the pressure factor is a maximum (slope is maximum), because a small ΔP_L, will produce a large change in flow, ΔQ.

- **Flow gain**
  - Flow gain was preciously defined as

$$G_{svf} = \frac{Flow}{Input\ Current}$$

Servo Analysis

- Now in this case, the input is not a current but a physical displacement of the spool.

$$G_{svf} = \frac{\Delta Q}{\Delta y}$$

Servo Analysis

Valve Stiffness

- Valve stiffness is found by installing pressure transducers at Ports A and B and measuring the load pressure, $P_L = P_A - P_B = \Delta P_L$, as the valve is opened. Spool travel is the displacement, y; therefore, valve stiffness is defined by

$$S_v = \frac{\Delta P_L}{\Delta y}$$

Servo Analysis

- Using the definition of pressure factor and flow gain, respectively, Eq11.59 may be rewritten as follows.

$$x = \frac{1}{As}[G_{svf}y + P_f P_L] - \lambda F$$
$$= \frac{G_{svf}}{As}[y + \frac{P_f}{G_{svf}} P_L] - \lambda F$$

Servo Analysis

- Load pressure is

$$P_L = F / A$$

- Substituting into Eq11.63

$$x = \frac{G_{svf}}{As}\left[y + \frac{P_f}{G_{svf}}\frac{F}{A}\right] - \lambda F$$

Servo Analysis

- To simplify, define

$$K_a = G_{svf} / A$$

- Using the expression for valve stiffness

$$S_v = \frac{\Delta P_L}{\Delta y} = \frac{\Delta Q / \Delta y}{\Delta Q / \Delta P_L} = \frac{G_{svf}}{P_f}$$

Servo Analysis

- Eq11.64 becomes

$$x = \frac{K_a}{s}(y + \frac{F}{S_v A}) - \lambda F$$

- Defining

$$K_b = -S_v A$$

Servo Analysis

- On substituting

$$x = \frac{K_a}{s}(y - \frac{F}{K_b}) - \lambda F \text{.............}(11.67)$$

- Typically the load will have mass, elasticity and energy dissipation characteristics, so the equation of motion is

$$F = m\ddot{x} + c\dot{x} + kx$$

Servo Analysis

- In the Laplace domain,

$$F = (ms^2 + cs + k)x$$

- Adverse stability conditions arises when the load is primarily an inertia load. Neglecting elasticity (k=0) and energy dissipation (c = 0),

$$F = ms^2 x \text{.................}(11.70)$$

Servo Analysis

- Substitution of Eq.(11.70) into Eq(11.67),

$$x = \frac{K_a}{s}\left(y - \frac{ms^2 x}{K_b}\right) - \lambda m s^2 x$$

$$x\left(\lambda m s^2 + \frac{K_a m s}{K_b} + 1\right) = \frac{K_a}{s} y$$

$$\frac{x}{y} = \frac{K_a}{S\left[\lambda m s^2 + (K_a m / K_b) s + 1\right]}$$

Servo Analysis

- The transfer function presented is appropriate for simple control systems using a servo cylinder.
- When studying more complex systems the transfer function will likewise be more complex.

End of Chapter 11

Thank You