

# Springs

## Chapter 19

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### Springs

- A spring is a flexible element used to exert a force or a torque and, at the same time, to store energy.
- The force can be a linear push or pull, or it can be radial, acting similarly to a rubber band around a roll of drawings.
- The torque can be used to cause a rotation, for example, to close a door on a cabinet or to provide a counterbalance force for a machine element pivoting on a hinge.

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**TABLE 19-1** Types of springs

Uses	Types of springs
Push	Helical compression spring
	Belleville spring
	Torsion spring: force acting at the end of the torque arm
Pull	Flat spring, such as a cantilever or leaf spring
	Helical extension spring
	Torsion spring: force acting at the end of the torque arm
	Flat spring, such as a cantilever or leaf spring
	Drawbar spring (special case of the compression spring)
Radial	Constant-force spring
	Garter spring, elastomeric band, spring clamp
Torque	Torsion spring, power spring

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**Kinds of Springs**

- Springs can be classified according to the direction and the nature of the force exerted by the spring when it is deflected.
- Springs are classified as push, pull, radial, and torsion.
- Helical compression springs are typically made from round wire, wrapped into a straight, cylindrical form with a constant pitch between adjacent coils.
  - Square or rectangular wire may also be used

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**Kinds of Springs con't**

- Without an applied load, the spring's length is called the free length.
- When a compression force is applied, the coils are pressed more closely together until they all touch, at which time the length is the minimum possible called the solid length.

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**Kinds of Springs con't**

- Helical extension springs appear to be similar to compression springs, having a series of coils wrapped into a cylindrical form.
- However, in extension springs, the coils either touch or are closely spaced under the no-load condition.

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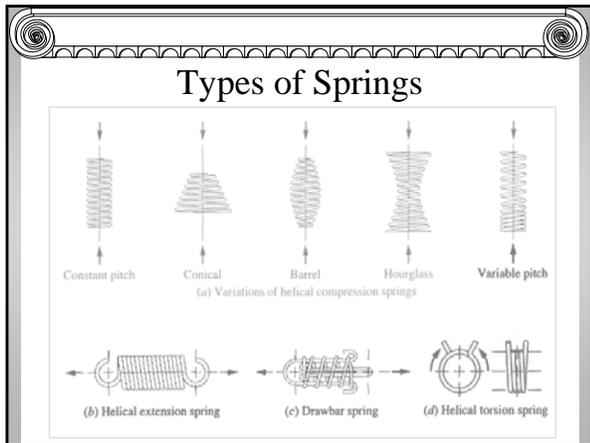
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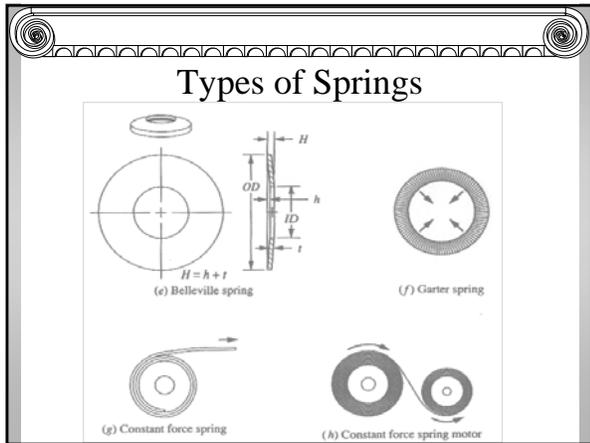
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Type	End configurations
Twist loop or hook	
Cross center loop or hook	
Side loop or hook	
Extended hook	
Special ends	

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### Kinds of Springs con't

- The drawbar spring incorporates a standard helical compression spring with two looped wire devices inserted through the inside of the spring.
- With such a design, a tensile force can be exerted by pulling on the loops while still placing the spring in compression.

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### Kinds of Springs con't

- A torsion spring is used to exert a torque as the spring is deflected by rotation about its axis.
- The common spring-action clothespin uses a torsion spring to provide the gripping action.
- Leaf springs are made from one or more flat strips of brass, bronze, steel, or other materials loaded as cantilevers or simple beams.
- They can provide a push or pull force as they are deflected from their free condition.
- Large forces can be exerted within a small space by leaf springs.

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### Kinds of Springs con't

- A Belleville spring has the shape of a shallow, conical disk with a central hole.
- It is sometimes called a Belleville washer because its appearance is similar to that of a flat washer.
- A very high spring rate or spring force can be developed in a small axial space with such springs.
- By varying the height of the cone relative to the thickness of the disk, the designer can obtain a variety of load-deflection characteristics.

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**Kinds of Springs con't**

- Garter springs are coiled wire formed into a continuous ring shape so that they exert a radial force around the periphery of the object to which they are applied.
- Either inward or outward forces can be obtained with different designs.

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**Kinds of Springs con't**

- Constant-force springs take the form of a coiled strip.
- The force required to pull the strip off the coil is virtually constant over a long length of pull.
- The magnitude of the force is dependent on the width, thickness, and radius of curvature of the coil and on the elastic modulus of the spring material.

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**Helical Compression Springs**

- In the most common form of helical compression spring, round wire is wrapped into a cylindrical form with a constant pitch between adjacent coils.
- This basic form is completed by a variety of end treatments.

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## Helical Compression Springs

- There are many uses of helical compression springs.
- The retractable ballpoint pen depends on the helical compression spring, usually installed around the ink supply barrel.
- Suspension systems for cars, trucks, and motorcycles frequently incorporate these springs.

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## Diameters

- The next figure shows the notation used in referring to the characteristic diameters of helical compression springs.
- The outside diameter (OD), the inside diameter (ID), and the wire diameter ( $D_w$ ) are obvious and can be measured with standard measuring instruments.

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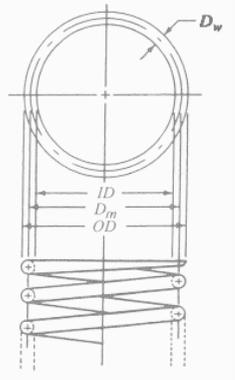
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**FIGURE 19-5**  
Notation for diameters



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## Diameters con't

- In calculating the stress and deflection of a spring, we use the mean diameter,  $D_m$ .
  - $OD = D_m + D_w$
  - $ID = D_m - D_w$
- The specification of the required wire diameter is one of the most important outcomes of the design of springs.
- Several types of materials are typically used for spring wire, and the wire is produced in sets of standard diameters covering a broad range.

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## Lengths

- It is important to understand the relationship between the length of the spring and the force exerted by it.
- The free length,  $L_f$ , is the length that the spring assumes when it is exerting no force.
- The solid length,  $L_s$ , is found when the spring is collapsed to the point where all the coils are touching.
- The spring is usually not compressed to the solid length during operation.

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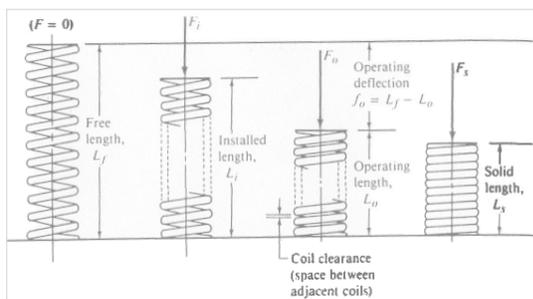
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## Notation for Lengths and Forces



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### Lengths con't

- The shortest length for the spring during normal operation is the operating length,  $L_o$ .
- A spring will be designed to operate between two limits of deflection.
- Consider the valve spring for an engine.
- When the valve is open, the spring assumes its shortest length, which is  $L_o$ .

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### Lengths con't

- Then, when the valve is closed, the spring gets longer but still exerts a force to keep the valve securely on its seat.
- The length at this condition is called the installed length,  $L_i$ .
- The valve spring length changes from  $L_o$  to  $L_i$  during normal operation as the valve itself reciprocates.

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### Forces

- Use the symbol  $F$  to indicate forces exerted by a spring, with various subscripts to specify which level of force is being considered.
  - $F_s$  = force at solid length,  $L_s$ ; the maximum force that the spring ever sees
  - $F_o$  = force at operating length,  $L_o$ ; the maximum force the spring sees in normal operation
  - $F_i$  = force at installed length,  $L_i$ ; the force varies between  $F_o$  and  $F_i$  for a reciprocating spring
  - $F_f$  = force at free length,  $L_f$ ; this force is zero

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## Spring Rate

- The relationship between the force exerted by a spring and its deflection is called its spring rate,  $k$ .
- Any change in force divided by the corresponding change in deflection can be used to compute the spring rate:
  - $k = \Delta F / \Delta L$
  - $k = (F_o - F_i) / (L_i - L_o)$
- If the spring rate is known, the force at any deflection can be computed.

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## Spring Index

- The ratio of the mean diameter of the spring to the wire diameter is called the spring index,  $C$ :
  - $C = D_m / D_w$
- It is recommended that  $C$  be greater than 5.0, with typical machinery springs having  $C$  values ranging from 5 to 12.
- For  $C$  less than 5, the forming of the spring will be difficult and the severe deformation required may create cracks in the wire.

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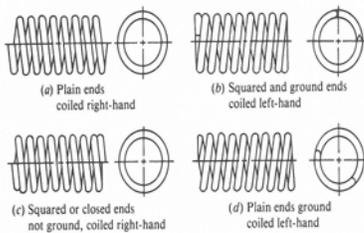
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## Spring End Conditions



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## Number of Coils

- The total number of coils in a spring will be called  $N$ .
- But in the calculation of stress and deflections for a spring, some of the coils are inactive and are neglected.
- For example, in a spring with squared and ground ends or simply squared ends, end coils are inactive, and the number of active coils,  $N_a$ , is  $N - 2$ .
- For plain ends, all coils are active:  $N_a = N$ .
- For plain coils with ground ends,  $N_a = N - 1$ .

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## Pitch

- Pitch,  $p$ , refers to the axial distance for a point on one coil to the corresponding point on the next adjacent coil.
- The relationships among the pitch, free length, wire diameter, and number of active coils are:
  - Squared and ground ends:  $L_f = pN_a + 2D_w$
  - Squared ends only:  $L_f = pN_a + 3D_w$
  - Plain and ground ends:  $L_f = p(N_a + 1)$
  - Plain ends:  $L_f = pN_a + D_w$

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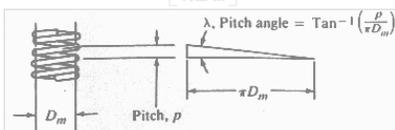
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## Pitch Angle

- This shows the pitch angle,  $\lambda$ ; note the larger the pitch angle, the steeper the coils appear to be.
- Most practical spring designs produce a pitch angle less than about  $12^\circ$ .
- If the angle is greater than  $12^\circ$ , undesirable compressive stresses develop in the wire, and the formulas used are inaccurate.

$$\lambda = \tan^{-1} \left( \frac{p}{\pi D_m} \right)$$




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### Installation Considerations

- Frequently, a spring is installed in a cylindrical hole or around a rod.
- When it is, adequate clearances must be provided.
- When a compression spring is compressed, its diameter gets larger.
- The inside diameter of a hole enclosing the spring must be greater than the outside diameter of the spring to eliminate rubbing.

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### Installation Considerations con't

- An initial diametral clearance of 1/10<sup>th</sup> of the wire diameter is recommended for springs having a diameter of 0.50 in (12mm) or greater.
- If a more precise estimate of the actual outside diameter of the spring is required, this can be used for the OD at the solid length condition:

$$OD_s = \sqrt{D_m^2 + \frac{P^2 - D_w^2}{\pi^2}} + D_w$$


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### Installation Considerations con't

- Even though the spring ID gets larger, it is also recommended that the clearance at the ID be approximately 0.1D<sub>w</sub>.

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### Coil Clearance

- The term coil clearance refers to the space between adjacent coils when the spring is compressed to its operating length,  $L_o$ .
- The actual coil clearance,  $cc$ , can be estimated from:
  - $cc = (L_o - L_s) / N_a$
- Our guideline is that the coil clearance should be greater than  $D_w / 10$ , especially in springs loaded cyclically.
- Another recommendation relates to the overall deflection of the spring:
  - $(L_o - L_s) > 0.15 (L_f - L_s)$

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### Stresses and Deflection

- As a compressive spring is compressed under an axial load, the wire is twisted.
- Therefore, the stress developed in the wire is torsional shear stress, and it can be derived from the classical equation  $\tau = Tc / J$ .
- When the equation is applied specifically to a helical compression spring, some modifying factors are needed to account for the curvature of the spring wire and for the direct shear stress created as the coils resist the vertical load.

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### Stresses and Deflection

- The resulting equation for stress is attributed to Wahl.
- The maximum shear stress, which occurs at the inner surface of the wire, is:

$$\tau = \frac{8KFD_m}{\pi D_w^3} = \frac{8KFC}{\pi D_w^3}$$

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## Deflection

- Because the primary manner of loading on the wire of a helical compression spring is torsion, the deflection is computed from the angle of twist formula:
  - $\theta = TL / GJ$
  - Where  $\theta$  = angle of twist in radians
  - T = applied torque
  - L = length of the wire
  - G = modulus of elasticity of the material in shear
  - J = polar moment of inertia of the wire

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## Deflection con't

- Use a different form of the equation in order to calculate the linear deflection,  $f$ , of the spring from the typical design variables of the spring:
 
$$f = \frac{8FD_m^3 N_a}{GD_w^4} = \frac{8FC^3 N_a}{GD_w^4}$$
- Recall that  $N_a$  is the number of active coils.
- The next table lists the values for G for typical spring materials.
- The wire diameter has a strong effect on the performance of the spring.

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TABLE 19-4 Spring wire modulus of elasticity in shear ( $G$ ) and tension ( $E$ )

Material and ASTM no.	Shear modulus, $G$		Tension modulus, $E$	
	(psi)	(GPa)	(psi)	(GPa)
Hard-drawn steel: A227	$11.5 \times 10^6$	79.3	$28.6 \times 10^6$	197
Music wire: A228	$11.85 \times 10^6$	81.7	$29.0 \times 10^6$	200
Oil-tempered: A229	$11.2 \times 10^6$	77.2	$28.5 \times 10^6$	196
Chromium-vanadium: A231	$11.2 \times 10^6$	77.2	$28.5 \times 10^6$	196
Chromium-silicon: A401	$11.2 \times 10^6$	77.2	$29.5 \times 10^6$	203
Stainless steels: A313				
Types 302, 304, 316	$10.0 \times 10^6$	69.0	$28.0 \times 10^6$	193
Type 17-7 PH	$10.5 \times 10^6$	72.4	$29.5 \times 10^6$	203
Spring brass: B134	$5.0 \times 10^6$	34.5	$15.0 \times 10^6$	103
Phosphor bronze: B159	$6.0 \times 10^6$	41.4	$15.0 \times 10^6$	103
Beryllium copper: B197	$7.0 \times 10^6$	48.3	$17.0 \times 10^6$	117
Monel and K-Monel	$9.5 \times 10^6$	65.5	$26.0 \times 10^6$	179
Inconel and Inconel-X	$10.5 \times 10^6$	72.4	$31.0 \times 10^6$	214

Note: Data are average values. Slight variations with wire size and treatment may occur.

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## Buckling

- The tendency for a spring to buckle increases as the spring becomes tall and slender, much as for a column.
- The next figure shows plots of the critical ratio of deflection to the free length versus the ratio of free length to the mean diameter for the spring.
- Three different end conditions are described in the figure.

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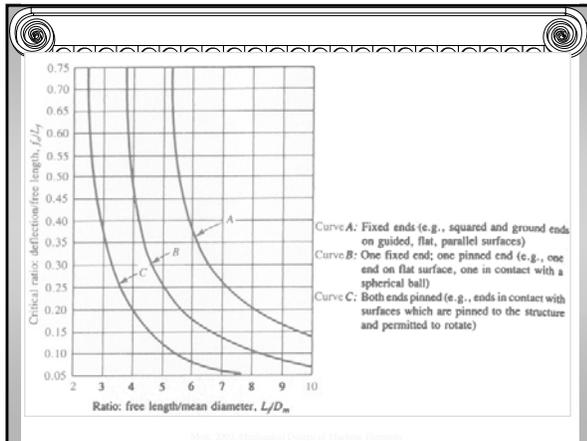
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## Buckling con't

- As an example of the use of that figure, consider a spring having squared and ground ends, a free length of 6.0 in, and a mean diameter of 0.75 in.
- Compute the deflection that would cause the spring to buckle:
  - $L_f / D_m = 6.0 / 0.75 = 8.0$
- Then, the critical deflection ratio is 0.20.
- From this compute the critical deflection:
  - $f_o / L_f = 0.20$  or  $f_o = 0.20(L_f) = 0.20(6.0 \text{ in}) = 1.20 \text{ in}$
- That is, if the spring is deflected more than 1.20 in, the spring should buckle.

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## Analysis

✦ This section demonstrates the use of the concepts developed in previous sections to analyze the geometry and the performance characteristics of a given spring.

✦ Example 19-1 illustrates the procedure.

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A spring is known to be made from music wire, ASTM A228 steel, but no other data are known. You are able to measure the following features using simple measurement tools:

- Free length =  $L_f = 1.75$  in
- Outside diameter =  $OD = 0.561$  in
- Wire diameter =  $D_w = 0.055$  in
- The ends are squared and ground.
- The total number of coils is 10.0.

This spring will be used in an application where the normal operating load is to be 14.0 lb. Approximately 300 000 cycles of loading are expected.

For this spring, compute and/or do the following:

1. The music wire gage number, mean diameter, inside diameter, spring index, and Wahl factor.
2. The expected stress at the operating load of 14.0 lb.
3. The deflection of the spring under the 14.0-lb load.
4. The operating length, solid length, and spring rate.
5. The force on the spring when it is at its solid length and the corresponding stress at solid length.
6. The design stress for the material; then compare it with the actual operating stress.
7. The maximum permissible stress; then compare it with the stress at solid length.
8. Check the spring for buckling and coil clearance.
9. Specify a suitable diameter for a hole in which to install the spring.

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The solution is presented in the same order as the requested items just listed. The formulas used are found in the preceding sections of this chapter.

**Step 1.** The wire is 24-gage music wire (Table 19-2). Thus,

$$D_m = OD - D_w = 0.561 - 0.055 = 0.506 \text{ in}$$

$$ID = D_m - D_w = 0.506 - 0.055 = 0.451 \text{ in}$$

$$\text{Spring index} = C = D_m/D_w = 0.506/0.055 = 9.20$$

$$\text{Wahl factor} = K = (4C - 1)/(4C - 4) + 0.615/C$$

$$K = [4(9.20) - 1]/[4(9.20) - 4] + 0.615/9.20$$

$$K = 1.158$$

**Step 2.** Stress in spring at  $F = F_o = 14.0$  lb [Equation (19-4)]:

$$\tau_o = \frac{8KF_oC}{\pi D_w^2} = \frac{8(1.158)(14.0)(9.20)}{\pi(0.055)^2} = 125\,560 \text{ psi}$$

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**Step 3.** Deflection at operating force [Equation (19-6)]:

$$f_o = \frac{8F_o C^3 N_o}{GD_w} = \frac{8(14.0)(9.20)^3(8.0)}{(11.85 \times 10^6)(0.055)} = 1.071 \text{ in}$$

Note that the number of active coils for a spring with squared and ground ends is  $N_o = N - 2 = 10.0 - 2 = 8.0$ . Also, the spring wire modulus,  $G$ , was found in Table 19-4. The value of  $f_o$  is the deflection from *free length* to the operating length.

**Step 4.** Operating length: We compute operating length as

$$L_o = L_f - f_o = 1.75 - 1.071 = 0.679 \text{ in}$$

$$\text{Solid length} = L_s = D_w(N) = 0.055(10.0) = 0.550 \text{ in}$$

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Spring Index: We use Equation (19-1).

$$k = \frac{\Delta F}{\Delta L} = \frac{F_o}{L_f - L_o} = \frac{F_o}{f_o} = \frac{14.0 \text{ lb}}{1.071 \text{ in}} = 13.07 \text{ lb/in}$$

**Step 5.** We can find the force at solid length by multiplying the spring rate times the deflection from the free length to the solid length. Then

$$F_s = k(L_f - L_s) = (13.07 \text{ lb/in})(1.75 \text{ in} - 0.550 \text{ in}) = 15.69 \text{ lb}$$

The stress at solid length,  $\tau_s$ , could be found from Equation (19-4), using  $F = F_s$ . However, an easier method is to recognize that the stress is directly proportional to the force on the spring and that all of the other data in the formula are the same as those used to compute the stress under the operating force,  $F_o$ . We can then use the simple proportion

$$\tau_s = \tau_o (F_s/F_o) = (125\,560 \text{ psi})(15.69/14.0) = 140\,700 \text{ psi}$$

**Step 6.** Design stress,  $\tau_d$ : From Figure 19-9, in the graph of design stress versus spring wire diameter for ASTM A228 steel, we can use the *average service* curve based on the expected number of cycles of loading. We read  $\tau_d = 135\,000 \text{ psi}$  for the 0.055-in wire. Because the actual operating stress,  $\tau_o$ , is less than this value, it is satisfactory.

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**Step 7.** Maximum allowable stress,  $\tau_{\max}$ : It is recommended that the *light service* curve be used to determine this value. For  $D_w = 0.055$ ,  $\tau_{\max} = 150\,000 \text{ psi}$ . The actual expected maximum stress that occurs at solid length ( $\tau_s = 140\,700 \text{ psi}$ ) is less than this value, and therefore the design is satisfactory with regard to stresses.

**Step 8.** Buckling: To evaluate buckling, we must compute

$$L_f/D_w = (1.75 \text{ in})/(0.056 \text{ in}) = 3.46$$

Referring to Figure 19-15 and using curve A for squared and ground ends, we see that the critical deflection ratio is very high and that buckling should not occur. In fact, for any value of  $L_f/D_w < 5.2$ , we can conclude that buckling will not occur.

Coil clearance,  $cc$ : We evaluate  $cc$  as follows:

$$cc = (L_o - L_s)/N_o = (0.679 - 0.550)/(8.0) = 0.016 \text{ in}$$

Comparing this to the recommended minimum clearance of

$$D_w/10 = (0.055 \text{ in})/10 = 0.0055 \text{ in}$$

we can judge this clearance to be acceptable.

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*Step 9.* Hole diameter: It is recommended that the hole into which the spring is to be installed should be greater in diameter than the *OD* of the spring by the amount of  $D_w/10$ . Then

$$D_{\text{hole}} > OD + D_w/10 = 0.561 \text{ in} + (0.055 \text{ in})/10 = 0.567 \text{ in}$$

A diameter of 5/8 in (0.625 in) would be a satisfactory standard size.  
This completes the example problem.

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## Design of Helical Compression Springs

➤ The objective of the design of helical compression springs is to specify the geometry for the spring to operate under specified limits of load and deflection, possibly with space limitations, also.

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## Example Problem 2

A helical compression spring is to exert a force of 8.0 lb when compressed to a length of 1.75 in. At a length of 1.25 in, the force must be 12.0 lb. The spring will be installed in a machine that cycles slowly, and approximately 200 000 cycles total are expected. The temperature will not exceed 200°F. The spring will be installed in a hole having a diameter of 0.75 in.

For this application, specify a suitable material, wire diameter, mean diameter, *OD*, *ID*, free length, solid length, number of coils, and type of end condition. Check the stress at the maximum operating load and at the solid length condition.

The first of two solution procedures will be shown. The numbered steps can be used as a guide for future problems and as a kind of algorithm for the spreadsheet approach that follows the manual solution.

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The procedure works directly toward the overall geometry of the spring by specifying the mean diameter to meet the space limitations. The process requires that the designer have tables of data available for wire diameters (such as Table 19-2) and graphs of design stresses for the material from which the spring will be made (such as Figures 19-8 through 19-13). We must make an initial estimate for the design stress for the material by consulting the charts of design stress versus wire diameter to make a reasonable choice. In general, more than one trial must be made, but the results of early trials will help you decide the values to use for later trials.

**Step 1.** Specify a material and its shear modulus of elasticity,  $G$ .

For this problem, several standard spring materials can be used. Let's select ASTM A231 chromium-vanadium steel wire, having a value of  $G = 11\,200\,000$  psi (see Table 19-4).

**Step 2.** From the problem statement, identify the operating force,  $F_o$ ; the operating length at which that force must be exerted,  $L_o$ ; the force at some other length, called the *installed force*,  $F_i$ ; and the installed length,  $L_i$ .

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Remember,  $F_o$  is the maximum force that the spring experiences under normal operating conditions. Many times, the second force level is not specified. In that case, let  $F_i = 0$ , and specify a design value for the free length,  $L_f$ , in place of  $L_i$ .

For this problem,  $F_o = 12.0$  lb;  $L_o = 1.25$  in;  $F_i = 8.0$  lb; and  $L_i = 1.75$  in.

**Step 3.** Compute the spring rate,  $k$ , using Equation (19-1a):

$$k = \frac{F_o - F_i}{L_i - L_o} = \frac{12.0 - 8.0}{1.75 - 1.25} = 8.00 \text{ lb/in}$$

**Step 4.** Compute the free length,  $L_f$ :

$$L_f = L_i + F_i/k = 1.75 \text{ in} + [(8.00 \text{ lb})/(8.00 \text{ lb/in})] = 2.75 \text{ in}$$

The second term in the preceding equation is the amount of deflection from free length to the installed length in order to develop the installed force,  $F_i$ . Of course, this step becomes unnecessary if the free length is specified in the original data.

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**Step 5.** Specify an initial estimate for the mean diameter,  $D_m$ .

Keep in mind that the mean diameter will be smaller than the  $OD$  and larger than the  $ID$ . Judgment is necessary to get started. For this problem, let's specify  $D_m = 0.60$  in. This should permit the installation into the 0.75-in-diameter hole.

**Step 6.** Specify an initial design stress.

The charts for the design stresses for the selected material can be consulted, considering also the service. In this problem, we should use average service. Then for the ASTM A231 steel, as shown in Figure 19-11, a nominal design stress would be 130 000 psi. This is strictly an estimate based on the strength of the material. The process includes a check on stress later.

**Step 7.** Compute the trial wire diameter by solving Equation (19-4) for  $D_w$ . Notice that everything else in the equation is known except the Wahl factor,  $K$ , because it depends on the wire diameter itself. But  $K$  varies only little over the normal range of spring indexes,  $C$ . From Figure 19-14, note that  $K = 1.2$  is a nominal value. This, too, will be checked later. With the assumed value of  $K$ , some simplification can be done:

$$D_w = \left[ \frac{8KF_o D_m}{\pi \tau_d} \right]^{1/3} = \left[ \frac{(8)(1.2)(F_o)(D_m)}{(\pi)(\tau_d)} \right]^{1/3}$$

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Combining constants gives

$$D_w = \left[ \frac{8KF_o D_m}{\pi \tau_d} \right]^{1/3} = \left[ \frac{(3.06)(F_o)(D_m)}{(\tau_d)} \right]^{1/3} \quad (19-7)$$

For this problem,

$$D_w = \left[ \frac{(3.06)(F_o)(D_m)}{\tau_d} \right]^{1/3} = \left[ \frac{(3.06)(12)(0.6)}{130\,000} \right]^{0.333}$$

$$D_w = 0.0553 \text{ in}$$

**Step 8.** Select a standard wire diameter from the tables, and then determine the design stress and the maximum allowable stress for the material at that diameter. The design stress will normally be for average service, unless high cycling rates or shock indicate that severe service is warranted. The light service curve should be used with care because it is very near to the yield strength. In fact, we will use the light service curve as an estimate of the maximum allowable stress.

For this problem, the next larger standard wire size is 0.0625 in, no. 16 on the U.S. Steel Wire Gage chart. For this size, the curves in Figure 19-11 for ASTM A231 steel wire show the design stress to be approximately 145 000 psi for average service, and the maximum allowable stress to be 170 000 psi from the light service curve.

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**Step 9.** Compute the actual values of  $C$  and  $K$ , the spring index and the Wahl factor:

$$C = \frac{D_m}{D_w} = \frac{0.60}{0.0625} = 9.60$$

$$K = \frac{4C - 1}{4C - 4} + \frac{0.615}{C} = \frac{4(9.60) - 1}{4(9.60) - 4} + \frac{0.615}{9.60} = 1.15$$

**Step 10.** Compute the actual expected stress due to the operating force,  $F_o$ , from Equation (19-4):

$$\tau_o = \frac{8KF_o D_m}{\pi D_w^3} = \frac{(8)(1.15)(12.0)(0.60)}{(\pi)(0.0625)^3} = 86\,450 \text{ psi}$$

Comparing this with the design stress of 145 000 psi, we see that it is safe.

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**Step 11.** Compute the number of active coils required to give the proper deflection characteristics for the spring. Using Equation (19-6) and solving for  $N_o$ , we have

$$f = \frac{8F_o C N_o}{GD_w}$$

$$N_o = \frac{fGD_w}{8FC^3} = \frac{GD_w}{8kC^3} \quad (\text{Note: } F/f = k, \text{ the spring rate.}) \quad (19-8)$$

Then, for this problem,

$$N_o = \frac{GD_w}{8kC^3} = \frac{(11\,200\,000)(0.0625)}{(8)(8.0)(9.60)^3} = 12.36 \text{ coils}$$

Notice that  $k = 8.0 \text{ lb/in}$  is the spring rate. Do not confuse this with  $K$ , the Wahl factor.

**Step 12.** Compute the solid length,  $L_s$ ; the force on the spring at solid length,  $F_s$ ; and the stress in the spring at solid length,  $\tau_s$ . This computation will give the maximum stress that the spring will receive.

The solid length occurs when all of the coils are touching, but recall that there are two inactive coils for springs with squared and ground ends. Thus,

$$L_s = D_w(N_o + 2) = 0.0625(14.36) = 0.898 \text{ in}$$

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The force at solid length is the product of the spring rate times the deflection to solid length ( $L_f - L_s$ ):

$$F_s = k(L_f - L_s) = (8.0 \text{ lb/in})(2.75 - 0.898) \text{ in} = 14.8 \text{ lb}$$

Because the stress in the spring is directly proportional to the force, a simple method of computing the solid length stress is

$$\tau_s = (\tau_o)(F_s/F_o) = (86\,450 \text{ psi})(14.8/12.0) = 106\,750 \text{ psi}$$

When this value is compared with the maximum allowable stress of 170 000 psi, we see that it is safe, and the spring will not yield when compressed to solid length.

**Step 13.** Complete the computations of geometric features, and compare them with space and operational limitations:

$$OD = D_m + D_w = 0.60 + 0.0625 = 0.663 \text{ in}$$

$$ID = D_m - D_w = 0.60 - 0.0625 = 0.538 \text{ in}$$

These dimensions are satisfactory for installation in a hole having a diameter of 0.75 in.

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**Step 14.** The tendency to buckle is checked, along with the coil clearance.

This procedure completes the design of one satisfactory spring for this application. It may be desirable to make other trials to attempt to find a more nearly opti-

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## Extension Springs

- Extension springs are designed to exert a pulling force and to store energy.
- They are made from closely coiled helical coils similar in appearance to helical compression springs.
- Most extension springs are made with adjacent coils touching in such a manner that an initial force must be applied to separate the coils.
- Once the coils are separated, the force is linearly proportional to the deflection, as it is for helical compression springs.

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### Extension Springs con't

- The stresses and deflections for an extension spring can be computed by using the formulas used for compression springs.
- $\tau = \frac{8FD_s}{\pi D^3} + \frac{8FC}{\pi D^3}$  is used for the torsional shear stress,  $K = \frac{4C-1}{4C+1} + \frac{0.615}{C}$  for the Wahl factor to account for the curvature of the wire and the direct shear stress, and  $\delta = \frac{8FD_s N_s}{G D^4} + \frac{8FC N_s}{G D^4}$  for the deflection characteristics.

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### Extension Springs con't

- All coils in an extension spring are active.
- In addition, since the end loops or hooks deflect, their deflection may affect the actual spring rate.
- The initial tension in an extension spring is typically 10% to 25% of the maximum design force.

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### End Configurations

- A wide variety of end configurations may be obtained for attaching the spring to mating machine elements, some of which are shown in the next slide.
- The cost of the spring can be greatly affected by its end type, and it is recommended that the manufacturer be consulted before ends are specified.

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Type	End configurations
Twist loop or hook	
Cross center loop or hook	
Side loop or hook	
Extended hook	
Special ends	

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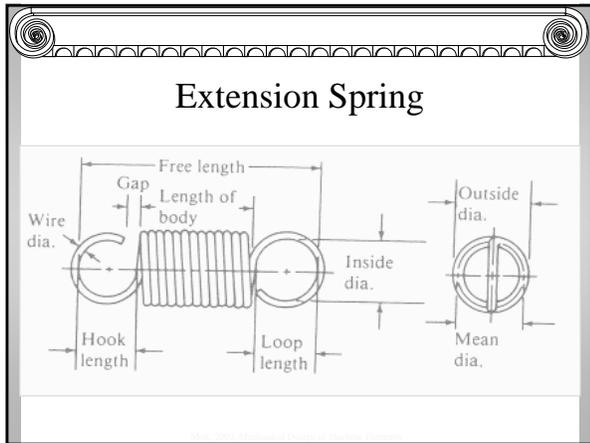
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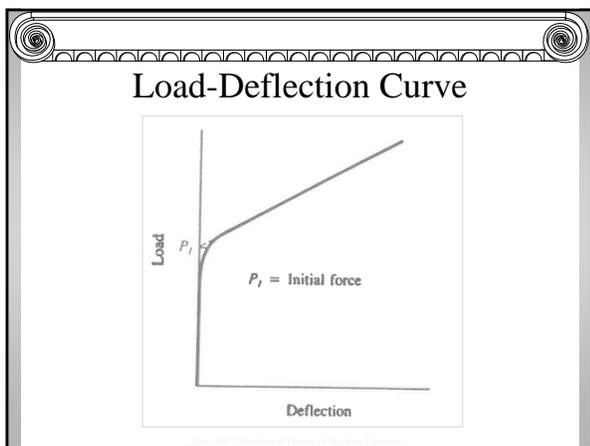
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### End Configurations con't

- Frequently, the weakest part of an extension spring is its end, especially in fatigue loading cases.
- The loop end, for example, has a high bending stress at point A and a torsional shear stress at point B.

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### End Configurations con't

(a) Bending stress at A      (b) Torsion stress at B

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### End Configurations con't

➤ Approximations for the stresses at these points can be computed as follows:

**Bending Stress at A**

$$\sigma_A = \frac{16D_m F_o K_1}{\pi D_w^3} + \frac{4F_o}{\pi D_w^2}$$

$$K_1 = \frac{4C_1^2 - C_1 - 1}{4C_1(C_1 - 1)}$$

$$C_1 = 2R_1/D_w$$


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