Shaft Design

Chapter 12

Material taken from Mott, 2003, Machine Elements in Mechanical Design

Shaft Design

- A shaft is the component of a mechanical device that transmits rotational motion and power.
- It is integral to any mechanical system in which power is transmitted from a prime mover, such as an electric motor or an engine, to other rotating parts of the system.

Shaft Design Procedure

- Because of the simultaneous occurrence of torsional shear and normal stresses due to bending, the stress analysis of a shaft virtually always involves the use of a combined stress approach.
- The recommended approach for shaft design and analysis is the distortion energy theory of failure.

Shaft Design Procedure

- Vertical shear stresses and direct normal stresses due to axial loads may also occur.
- On very short shafts or on portions of shafts where no bending or torsion occurs, such stresses may be dominant.

Procedure

1. Determine the rotational speed of the shaft.
2. Determine the power or the torque to be transmitted by the shaft.
3. Determine the design of the power-transmitting components or other devices that will be mounted on the shaft, and specify the required location of each device.

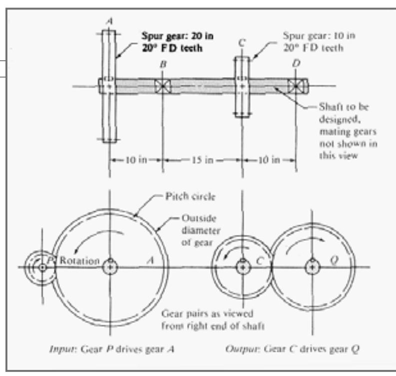
Procedure con't

4. Specify the location of bearings to support the shaft. Normally only two bearings are used to support a shaft. The reactions on bearings supporting radial loads are assumed to act at the midpoint of the bearings.
   - Bearings should be placed on either side of the power-transmitting elements if possible to provide stable support for the shaft and to produce reasonably well-balanced loading of the bearings.

Procedure con't

5. Propose the general form of the geometry for the shaft, considering how each element on the shaft will be held in position axially and how power transmission from each element to the shaft is to take place.

Intermediate Shaft



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Procedure con't

6. Determine the magnitude of torque that the shaft sees at all points.
   - It is recommended that a torque diagram be prepared.
7. Determine the forces that are exerted on the shaft, both radially and axially.

Procedure con't

8. Resolve the radial forces into components in perpendicular directions, usually vertically and horizontally.
9. Solve for the reactions on all support bearings in each plane.
10. Produce the complete shearing force and bending moment diagrams to determine the distribution of bending moments in the shaft.

Procedure con't

11. Select the material from which the shaft will be made, and specify its condition: cold-drawn, heat-treated, etc
    - Plain carbon or alloy steels with medium carbon content are typical, such as AISI 1040, 4140, 4340, 4660, 5150, 6150, and 8650.
    - Good ductility with percent elongation above about 12% is recommended.
    - Determine the ultimate strength, yield strength, and percent elongation of the selected material.

Procedure con't

12. Determine an appropriate design stress, considering the manner of loading
    - Smooth
    - Shock
    - Repeated and reversed
    - Other

Procedure con’t

13. Analyze each critical region of the shaft to determine the minimum acceptable diameter of the shaft to ensure safety under the loading at that point.
    - In general, the critical points are several and include those where a change of diameter takes place, where the higher values of torque and bending moment occur, and where stress concentrations occur.

Procedure con’t

14. Specify the final dimensions for each point on the shaft.
    - Design details such as tolerances, fillet radii, shoulder heights, and keyseat dimensions must also be specified.
    - Sometimes the size and the tolerance for a shaft diameter are dictated by the element to be mounted there.

Forces Exerted on Shafts

- Gears, belt sheaves, chain sprockets, and other elements typically carried by shafts exert forces on the shaft that cause bending moments.

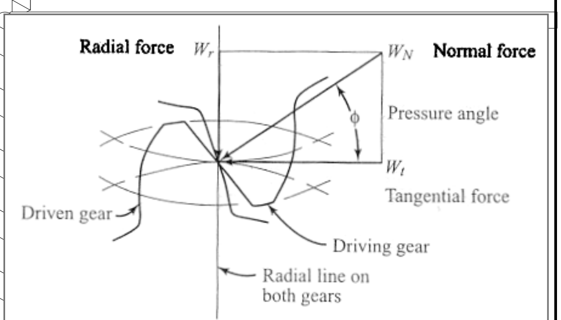
Spur Gears

- The force exerted on a gear tooth during power transmission acts normal (perpendicular) to the involute-tooth profile.
- It is convenient for the analysis of shafts to consider the rectangular components of this force acting in the radial and tangential directions.

Spur Gears con't

- It is most convenient to compute the tangential force, W_t, directly from the known torque being transmitted by the gear.
  - $T = 63000\ (P)/n$

Forces on Teeth of Driven Gear



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Tangential Force

- $W_t = T / (D/2)$
- Where P = power being transmitted in hp
- n = rotational speed in rpm
- T = Torque on the gear in lb*in
- D = pitch diameter of the gear in inches

Tangential Force con't

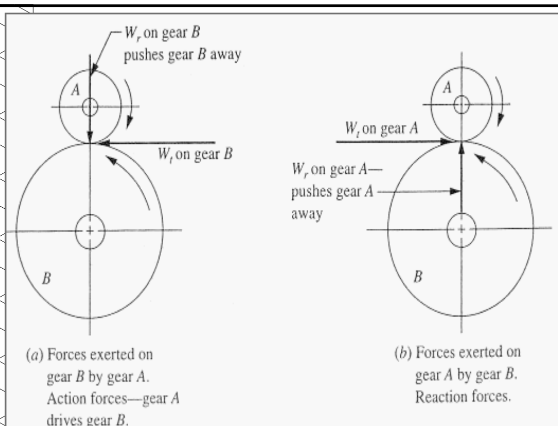
- The angle between the total force and the tangential component is equal to the pressure angle, ϕ, of the tooth form.
- So, if the tangential force is known, the radial force can be found from:
  - $W_r = W_t \tan \phi$

Tangential Force con't

- There is no need to compute the normal force.
- For gears, the pressure angle is typically 14 ½ º, 20º, or 25º.

Directions for Forces

- Representing the forces on gears in their correct directions is essential to an accurate analysis of forces and stresses in the shafts that carry the gears.
- The force system, shown next, represents the action of the driving gear A on the driven gear B.



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Directions for Forces

- The tangential force, W_t, pushes perpendicular to the radial line causing the driven gear to rotate.
- The radial force, W_r, exerted by the driving gear A, acts along the radial line tending to push the driven gear B away.

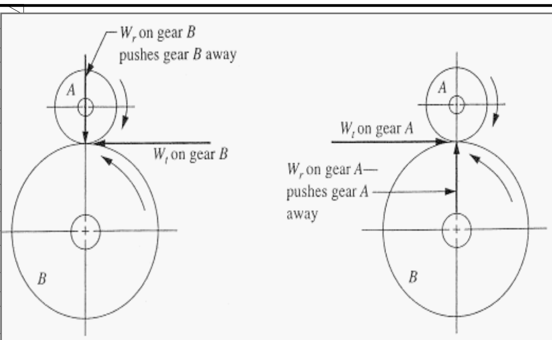
Directions for Forces

- ACTION:
- Driver pushes on driven gear
  - W_t: Acts to the left
  - W_r: acts downward

- REACTION:
- Driven gear pushes back on driver
  - W_t: acts to the right
  - W_r: acts upward

Directions for Forces

- Whenever you need to determine the direction of forces acting on a given gear, first determine whether it is a driver or driven gear.
- Then visualize the action forces of the driver.



(*a*) Forces exerted on gear *B* by gear *A*. Action forces—gear *A* drives gear *B*.

(*b*) Forces exerted on gear *A* by gear *B*. Reaction forces.

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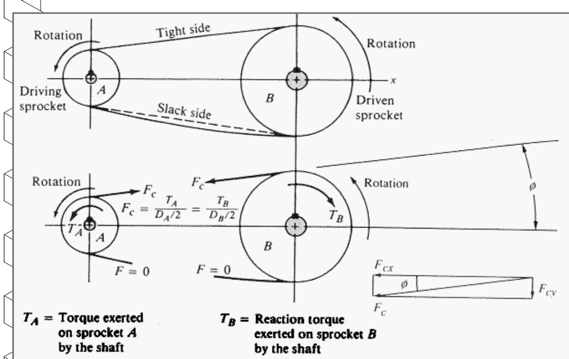
Directions for Forces

- If the gear of interest is the driven gear, these are the forces on it.
- If the gear of interest is the driver gear, the forces on it act in the opposite directions to the action forces.

Shows a pair of chain sprockets transmitting power



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Chain Sprockets

- The upper part of the chain is in tension and produces the torque on either sprocket.
- The lower part of the chain, or the slack side, exerts no force on either sprocket.
- Therefore, the total bending force on the shaft carrying the sprocket is equal to the tension in the tight side of the chain.

Chain Sprockets con't

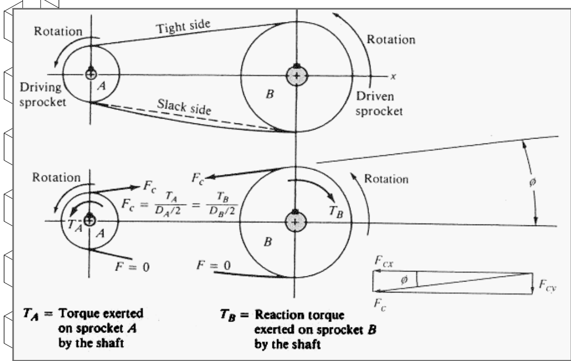
- If the torque on a certain sprocket is known,
  - $F_c = T / (D/2)$
  - Where D = pitch diameter of that sprocket.
- Notice that the force, F_c, acts along the direction of the tight side of the chain.

Chain Sprockets con't

- Because of the size difference between the two sprockets, that direction is at some angle from the centerline between the shaft centers.
- A precise analysis would call for the force, F_c, to be resolved into components parallel to the centerline and perpendicular to it.

Chain Sprockets con't

- $F_{cx} = F_c \cos \theta$
- $F_{cy} = F_c \sin \theta$
  - Where the x-direction is parallel to the centerline
  - The y-direction is perpendicular to it
  - The angle θ is the angle of inclination of the tight side of the chain with respect to the x-direction



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Chain Sprockets con't

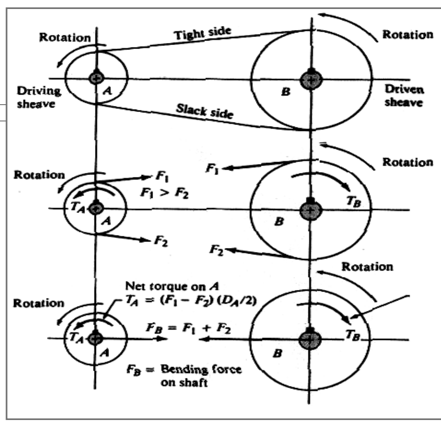
- These two components of the force would cause bending in both the x-direction and the y-direction.
- Alternatively, the analysis could be carried out in the direction of the force, F_c, in which single plane bending occurs.

Chain Sprockets con't

- If the angle is small, little error will result from the assumption that the entire force, F_c, acts along the x-direction.

V-Belt Sheaves

- The general appearance of the V-belt drive system looks similar to the chain drive system.
- There is one important difference: both sides of the V-belt are in tension, as shown in the next slide.



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V-Belt Sheaves con't

- The tight side tension, F_1, is greater than the "slack"side tension, F_2, and there is a net driving force on the sheaves equal to:
  - $F_N = F_1 - F_2$
- The magnitude of the net driving force can be computed from the torque transmitted:
  - $F_N = T / (D/2)$

V-Belt Sheaves con't

- Notice that the bending force on the shaft carrying the sheave is dependent on the sum, $F_1 + F_2 = F_B$.
- To be more precise, the components of F_1 and F_2 parallel to the line of centers of the two sprockets should be used.
- But unless the two sprockets are radically different in diameter, little error will result from $F_B = F_1 + F_2$.

V-Belt Sheaves con't

- To determine the bending force, F_B, a second equation involving the two forces F_1 and F_2 is needed. For V-belt drives, the ratio is:
  - $F_1 / F_2 = 5$
- It is convenient to derive a relationship between F_N and F_B of the form:
  - $F_B = CFN$
  - Where C = constant to be determined

V-Belt Sheaves con't

- But from $F_1 = 5F_2$,

V-Belt Sheaves con't

- Then, for V-belt drives:
- It is customary to consider the bending force, F_B, to act as a single force in the direction along the line of centers of the two sheaves.

Flat-Belt Pulleys

- The analysis of the bending force exerted on shafts by flat-belt pulleys is identical to that for V-belt sheaves except that the ratio of the tight side to the slack side tension is typically taken to be 3 instead of 5.
- Using the same logic as with V-belt sheaves, compute the constant C to be 2.0.
- Then, for flat-belt drives,
  - $F_B = 2.0F_N = 2.0T / (D/2)$

Stress Concentrations

- In order to mount and locate the several types of machine elements on shafts properly, a final design typically contains several diameters, keyseats, ring grooves, and other geometric discontinuities that create stress concentrations.

Stress Concentrations

- These stress concentrations must be taken into account during the design analysis.
- But a problem exists because the true design values of the stress concentration factors, K_t, are unknown at the start of the design process.

Stress Concentrations

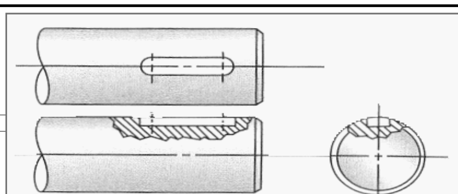
- Most of the values are dependent on the diameters of the shaft and on the fillet and groove geometries, and these are the objectives of the design.

Preliminary Design Values for K_t

- Considered here are the types of geometric discontinuities most often found in power-transmitting shafts: keyseats, shoulder fillets, and retaining ring grooves.
- In each case, a suggested design value is relatively high in order to produce a conservative result for the first approximation to the design.
- Again it is emphasized that the final design should be checked for safety.

Keyseats

- A keyseat is a longitudinal groove cut into a shaft for the mounting of a key, permitting the transfer of torque from the shaft to a power-transmitting element, or vice versa.
- Two types of keyseats are most frequently used: profile and sled runner.



(a) Profile keyseat

(b) Sled runner keyseat

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Keyseats con't

- The profile keyseat is milled into the shaft, using an end mill having a diameter equal to the width of the key.
- The resulting groove is flat-bottomed and has a sharp, square corner at its end.
- The sled runner keyseat is produced by a circular milling cutter having a width equal to the width of the key.

Keyseats con’t

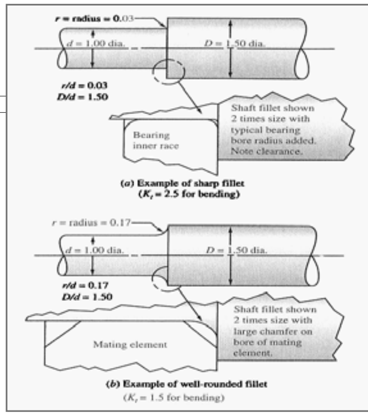
- As the cutter begins or ends the keyseat, it produces a smooth radius.
- For this reason, the stress concentration factor for the sled runner keyseat is lower than that for the profile keyseat.

Keyseats con’t

- Normally used design values are:
  - K_t = 2.0 (profile)
  - K_t = 1.6 (sled runner)

Shoulder Fillets

- When a change in diameter occurs in a shaft to create a shoulder against which to locate a machine element, a stress concentration dependent on the ratio of the two diameters and on the radius in the fillet is produced.



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Shoulder Fillets con't

- It is recommended that the fillet radius be as large as possible to minimize the stress concentration, but at times the design of the gear, bearing, or other element affects the radius that can be used.

Shoulder Fillets con't

- The term 'sharp' here does not mean truly sharp, without any fillet radius at all.
- Such a shoulder configuration would have a very high stress concentration factor and should be avoided.
- Instead, sharp describes a shoulder with a relatively small fillet radius.

Shoulder Fillets con't

- When an element with a large chamfer on its bore is located against the shoulder, or when nothing at all seats against the shoulder, the fillet radius can be much larger (well-rounded), and the corresponding stress concentration factor is smaller.
  - $K_t = 2.5$ (sharp fillet)
  - $K_t = 1.5$ (well-rounded fillet)

Retaining Ring Grooves

- Retaining rings are used for many types of locating tasks in shaft applications.
- The rings are installed in grooves in the shaft after the element to be retained is in place.
- The geometry of the groove is dictated by the ring manufacturer.

Retaining Ring Grooves

- Its usual configuration is a shallow groove with straight side walls and bottom and a small fillet at the base of the groove.
- The behavior of the shaft in the vicinity of the groove can be approximated by considering two sharp-filleted shoulders positioned close together.

Retaining Ring Grooves

- For preliminary design, apply K_t = 3.0 to the bending stress at a retaining ring groove to account for the rather sharp fillet radii.

Design Stresses for Shafts

- In a given shaft, several different stress conditions can exist at the same time.
- For any part of the shaft that transmits power, there will be a torsional shear stress, while bending stress is usually present on the same parts.

Design Stresses for Shafts con’t

- Only bending stresses may occur on other parts.
- Some points may not be subjected to either bending or torsion but will experience vertical shearing stress.
- Axial tensile or compressive stresses may be superimposed on the other stresses.
- Then there may be some points where no significant stresses at all are created.

Design Stresses for Shafts con't

- The decision of what design stress to use depends on the particular situation at the point of interest.
- In many shaft design and analysis projects, computations must be done at several points to account completely for the variety of loading and geometry conditions that exist.

Design Stresses for Shafts con't

- The bending stresses will be assumed to be completely reversed and repeated because of the rotation of the shaft.
- Because ductile materials perform better under such loads, it will be assumed that the material for the shaft is ductile and that the torsional loading is relatively constant and acting in one direction.

Design Shear Stress-Steady Torque

- The best predictor of failure in ductile materials due to a steady shear stress was the distortion energy theory in which the design shear stress is computed from:
- We will use this value for steady torsional shear stress, vertical shear stress, or direct shear stress in a shaft.

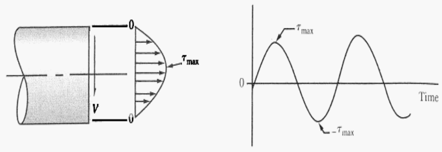
Design Shear Stress-Reversed Vertical Shear

- Points on a shaft where no torque is applied and where the bending moments are zero or very low are often subjected to significant vertical shearing forces which then govern the design analysis.
- This typically occurs where a bearing supports an end of a shaft and where no torque is transmitted in that part of the shaft.

Design Shear Stress-Reversed Vertical Shear

- The maximum shearing stress is at the neutral axis of the shaft.
- The stress decreases in a roughly parabolic manner to zero at the outer surface of the shaft.

FIGURE 12–9 Shearing stress in a rotating shaft due to vertical shearing force, V



(a) Distribution of shearing stress on a circular cross section of a shaft

(b) Variation of shearing stress on a given element at the surface of a circular rotating shaft

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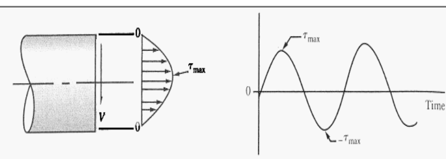
Design Shear Stress- Reversed Vertical Shear

- The maximum vertical shearing stress for the special case of a solid circular cross section can be computed from:
  - $\tau_{max} = 4V / 3A$
  - Where V = vertical shearing force
  - A = area of cross section

Design Shear Stress- Reversed Vertical Shear

- Where stress concentration factors are to be considered:
  - $\tau_{max} = K_t (4V / 3A)$
- Also note that the rotation of the shaft causes any point at the outer part of the cross section to experience a reversing shearing stress that varies from $+\tau_{max}$ to zero to $-\tau_{max}$ to zero in each revolution.

FIGURE 12–9 Shearing stress in a rotating shaft due to vertical shearing force, *V*



(*a*) Distribution of shearing stress on a circular cross section of a shaft

(*b*) Variation of shearing stress on a given element at the surface of a circular rotating shaft

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Design Shear Stress-Reversed Vertical Shear

- Then the stress analysis should be completed using a safety factor:
  - $N = s'_{sn} / \tau_{max}$
  - Where s'_{sn} is the endurance limit in shear
- Using the distortion energy theory. Then the endurance strength is:
  - $s'_{sn} = 0.577s'_n$
  - Where s'_n is the endurance limit of the material

Design Shear Stress-Reversed Vertical Shear

- Then the equation can be written in the form:
  - $N = 0.577s'_n / \tau_{max}$
- Expressed as a design stress:
  - $\tau_d = 0.577s'_n / N$
- Letting $\tau_{max} = \tau_d = [K_t(4V)] / 3A$ gives:

Design Shear Stress-Reversed Vertical Shear

- Solving for N gives:
- Solving for the required area:

Design Shear Stress-Reversed Vertical Shear

- By substituting:
  - $A = \pi D^2 / 4$
- Solve for D:

Design Shear Stress-Reversed Vertical Shear

- This equation should be used to compute the required diameter for a shaft where a vertical shearing force V is the only significant loading present.
- In most shafts, the resulting diameter will be much smaller than that required at other parts of the shaft where significant values of torque and bending occur.

Design Shear Stress-Reversed Vertical Shear

- Implementation of the previous equations has the complication that values for the stress concentration factor under conditions of vertical shearing stress are not well known.
- As an approximation, use the values for K_t for torsional stress when using these equations.

Design Shear Stress-Fatigue Loading

- For the repeated, reversed bending in a shaft caused by transverse loads applied to the rotating shaft, the design stress is related to the endurance strength of the shaft material.
- Refer to the discussion in Section 5-4 in Chapter 5 for the method of computing the estimated actual endurance strength, s'_n, for use in shaft design.

Design Shear Stress-Fatigue Loading

- Note that any stress concentration factor will be accounted for in the design equation developed later.
- Other factors, not considered here, that could have an adverse effect on the endurance strength of the shaft material are:
  - temperatures above 400ºF
  - variation in peak stress levels above the nominal endurance strength for some periods of time

Design Shear Stress-Fatigue Loading

- vibration
- residual stresses
- case hardening
- interference fits
- corrosion
- thermal cycling
- plating or surface coating
- stresses not accounted for in the basic stress analysis.

Design Shear Stress-Fatigue Loading

- For parts of the shaft subjected to only reversed bending, let the design stress be:
  - $\sigma_d = s'_n / N$

Design Factor

- Use N = 2.0 for typical shaft designs where there is average confidence in the data for material strength and loads.
- Higher values should be used for shock and impact loading and where uncertainty in the data exists.

Design Factor con’t

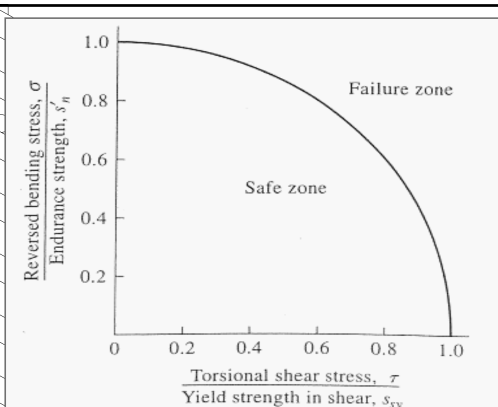
- Examples of shafts subjected to bending and torsion only are those carrying spur gears, V-belt sheaves, or chain sprockets.
- The power being transmitted causes the torsion, and the transverse forces on the elements cause bending.
- In the general case, the transverse forces do not all act in the same plane.

Design Factor con't

- In such cases, the bending moment diagrams for two perpendicular planes are prepared first.
- Then the resultant bending moment at each point of interest is determined.

Design Factor con't

- A design equation is now developed based on the assumption that the bending stress in the shaft is repeated and reversed as the shaft rotates, but that the torsional shear stress is nearly uniform.
- The design equation is based on the principle shown graphically in which the vertical axis is the ratio of the reversed bending stress to the endurance strength of the material.



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Design Factor con't

- The horizontal axis is the ratio of the torsional shear stress to the yield strength of the material in shear.
- The points having value of 1.0 on these axes indicated impending failure in pure bending or pure torsion, respectively.

Design Factor con't

- Experimental data show that failure under combinations of bending and torsion roughly follows the curve connecting these two points, which obeys the following equation:
  - $(\sigma / s'_n)^2 + (\tau / s_{ys})^2 = 1$

Design Factor con't

- We will use for the distortion energy theory.
- Also, a design factor can be introduced to each term on the left side of the equation to yield an expression based on design stresses:

Design Factor con't

- Now we can introduce a stress concentration factor for bending in the first term only, because this stress is repeated.
- No factor is needed for the torsional shear stress term because it is assumed to be steady, and stress concentrations have little or no effect on the failure potential:

Design Factor con't

- For rotating, solid, circular shafts, the bending stress due to a bending moment, M, is:
  - $\sigma = M / S$
  - Where $S = \pi D^3 / 32$ is the rectangular section modulus.
- The torsional shear stress is:
  - $\tau = T / Z_p$
  - Where $Z_p = \pi D^3 / 16$ is the polar section modulus
- Note that $Z_p = 2S$
  - $\tau = T / (2S)$

Design Factor con't

- Substituting these relationships:
- Now the terms N and S can be factored out, and the terms $\sqrt{3}$ and 2 can be brought outside the bracket in the torsional term:

Design Factor con't

- We now take the square root of the entire equation:
- Let $S = \pi D^3 / 32$ for a solid circular shaft:

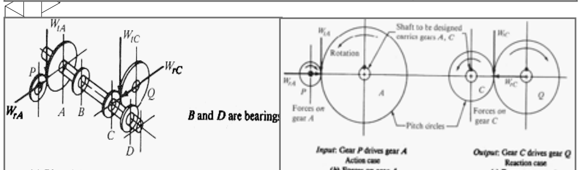
Design Factor con't

- Now we can solve for the diameter D:
- This is used for shaft design in this book. It is compatible with the standard ANSI B106.1M-1985. Note that it can also be used for pure bending or pure torsion.

Design the shaft shown in Figures 12–1 and 12–2. It is to be machined from AISI 1144 OQT 1000 steel. The shaft is part of the drive for a large blower system supplying air to a furnace. Gear A receives 200 hp from gear P. Gear C delivers the power to gear Q. The shaft rotates at 600 rpm.

First determine the properties of the steel for the shaft. From Figure A4–2, s_y = 83 000 psi, s_u = 118 000 psi, and the percent elongation is 19%. Thus, the material has good ductility. Using Figure 5–8, we can estimate s_n = 42 000 psi.

A size factor should be applied to the endurance strength because the shaft will be quite large to be able to carry 200 hp. Although we do not know the actual size at this time, we might select C_s = 0.75 from Figure 5–9 as an estimate.



(a) Pictorial view of forces on gears A and C
(b) Forces on gear A
(c) Forces on gear C

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A reliability factor should also be specified. This is a design decision. For this problem, let's design for a reliability of 0.99 and use $C_R = 0.81$. Now we can compute the modified endurance strength:

$$s_n' = s_n C_s C_R = (42\,000)(0.75)(0.81) = 25\,500 \text{ psi}$$

The design factor is taken to be $N = 2$. The blower is not expected to present any unusual shock or impact.

Now we can compute the torque in the shaft from Equation (12–1):

$$T = 63\,000(P)/n = 63\,000(200)/600 = 21\,000 \text{ lb}\cdot\text{in}$$

Note that only that part of the shaft from A to C is subjected to this torque. There is zero torque from the right of gear C over to bearing D.

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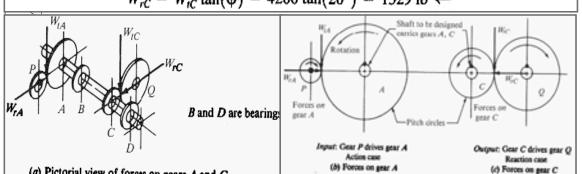
Forces on the Gears: Figure 12–11 shows the two pairs of gears with the forces acting *on gears* A *and* C *shown*. Observe that gear A is driven by gear P, and gear C drives gear Q. It is very important for the directions of these forces to be correct. The values of the forces are found from Equations (12–2) and (12–3).

$$W_{tA} = T_A/(D_A/2) = 21\,000/(20/2) = 2100 \text{ lb}\downarrow$$
$$W_{rA} = W_{tA}\tan(\phi) = 2100\tan(20°) = 764 \text{ lb} \rightarrow$$
$$W_{tC} = T_C/(D_C/2) = 21\,000/(10/2) = 4200 \text{ lb}\downarrow$$
$$W_{rC} = W_{tC}\tan(\phi) = 4200\tan(20°) = 1529 \text{ lb} \leftarrow$$



(a) Pictorial view of forces on gears A and C

Input: Gear P drives gear A
Action case
(b) Forces on gear A

Output: Gear C drives gear Q
Reaction case
(c) Forces on gear C

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Forces on the Shaft: The next step is to show these forces on the shaft in their proper planes of action and in the proper direction. The reactions at the bearings are computed, and the shearing force and bending moment diagrams are prepared. The results are shown in Figure 12–12.

We continue the design by computing the minimum acceptable diameter of the shaft at several points along the shaft. At each point, we will observe the magnitude of torque and the bending moment that exist at the point, and we will estimate the value of any stress concentration factors. If more than one stress concentration exist in the vicinity of the point of interest, the larger value is used for design. This assumes that the geometric discontinuities themselves do not interact, which is good practice. For example, at point A, the keyseat should end well before the shoulder fillet begins.

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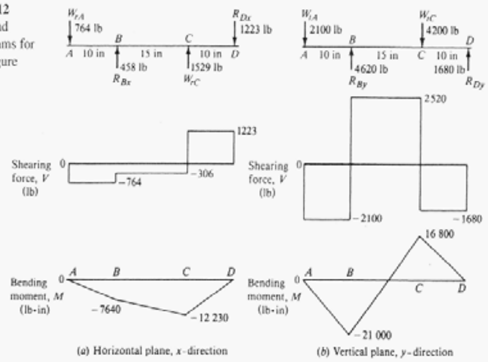
FIGURE 12–12 Load, shear, and moment diagrams for the shaft in Figure 12–10



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1. *Point A:* Gear *A* produces torsion in the shaft from *A* and to the right. To the left of *A*, where there is a retaining ring, there are no forces, moments, or torques.

The moment at *A* is zero because it is a free end of the shaft. Now we can use Equation (12–24) to compute the required diameter for the shaft at *A*, using only the torsion term.

$$D_1 = \left[\frac{32\,N}{\pi}\sqrt{\frac{3}{4}\left(\frac{T}{s_y}\right)^2}\right]^{1/3}$$

$$D_1 = \left[\frac{32(2)}{\pi}\sqrt{\frac{3}{4}\left(\frac{21\,000}{83\,000}\right)^2}\right]^{1/3} = 1.65 \text{ in}$$

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2. *Point B:* Point *B* is the location of a bearing with a sharp fillet to the right of *B* and a well-rounded fillet to the left. It is desirable to make D_2 at least slightly smaller than D_3 at the bearing seat to permit the bearing to be slid easily onto the shaft up to the place where it is pressed to its final position. There is usually a light press fit between the bearing bore and the shaft seat.

To the left of *B* (diameter D_2),

$$T = 21\,000 \text{ lb}\cdot\text{in}$$

The bending moment at *B* is the resultant of the moment in the *x*- and *y*-planes from Figure 12–12:

$$M_B = \sqrt{M_{Bx}^2 + M_{By}^2} = \sqrt{(7640)^2 + (21\,000)^2} = 22\,350 \text{ lb}\cdot\text{in}$$

$$K_t = 1.5 \text{ (well-rounded fillet)}$$

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Using Equation (12–24) because of the combined stress condition,

$$D_2 = \left[\left(\frac{32N}{\pi}\right)\sqrt{\left(\frac{K_t M}{s_n'}\right)^2 + \frac{3}{4}\left(\frac{T}{s_y}\right)^2}\right]^{1/3} \qquad \textbf{(12–24a)}$$

$$D_2 = \left[\frac{32(2)}{\pi}\sqrt{\left[\frac{1.5(22\,350)}{25\,500}\right]^2 + \frac{3}{4}\left[\frac{21\,000}{83\,000}\right]^2}\right]^{1/3} = 3.30 \text{ in}$$

At B and to the right of B (diameter D_3), everything is the same, except the value of $K_t = 2.5$ for the sharp fillet. Then

$$D_3 = \left[\frac{32(2)}{\pi}\sqrt{\left[\frac{2.5(22\,350)}{25\,500}\right]^2 + \frac{3}{4}\left[\frac{21\,000}{83\,000}\right]^2}\right]^{1/3} = 3.55 \text{ in}$$

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Notice that D_4 will be larger than D_3 in order to provide a shoulder for the bearing. Therefore, it will be safe. Its actual diameter will be specified after we have completed the stress analysis and selected the bearing at B. The bearing manufacturer's catalog will specify the minimum acceptable diameter to the right of the bearing to provide a suitable shoulder against which to seat the bearing.

3. *Point C:* Point C is the location of gear C with a well-rounded fillet to the left, a profile keyseat at the gear, and a retaining ring groove to the right. The use of a well-rounded fillet at this point is actually a design decision that requires that the design of the bore of the gear accommodate a large fillet. Usually this means that a chamfer is produced at the ends of the bore. The bending moment at C is

$$M_c = \sqrt{M_{Cx}^2 + M_{Cy}^2} = \sqrt{(12\,230)^2 + (16\,800)^2} = 20\,780 \text{ lb}\cdot\text{in}$$

To the left of C the torque of 21 000 lb·in exists with the profile keyseat giving $K_t = 2.0$. Then

$$D_5 = \left[\frac{32(2)}{\pi}\sqrt{\left[\frac{2.0(20\,780)}{25\,500}\right]^2 + \frac{3}{4}\left[\frac{21\,000}{83\,000}\right]^2}\right]^{1/3} = 3.22 \text{ in}$$

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To the right of C there is no torque, but the ring groove suggests $K_t = 3.0$ for design, and there is reversed bending. We can use Equation (12–24) with $K_t = 3.0$, $M = 20\,780$ lb·in and $T = 0$.

$$D_5 = \left[\frac{32(2)}{\pi}\sqrt{\left(\frac{(3.0)(20\,780)}{25\,500}\right)^2}\right]^{1/3} = 3.68 \text{ in}$$

Applying the ring groove factor of 1.06 raises the diameter to 3.90 in.

This value is higher than that computed for the left of C, so it governs the design at point C.

4. *Point D:* Point D is the seat for bearing D, and there is no torque or bending moment here. However, there is a vertical shearing force equal to the reaction at the bearing. Using the resultant of the x- and y-plane reactions, the shearing force is

$$V_D = \sqrt{(1223)^2 + (1680)^2} = 2078 \text{ lb}$$

We can use Equation (12–16) to compute the required diameter for the shaft at this point:

$$D = \sqrt{2.94\, K_t(V)N/s_n'} \qquad \textbf{(12–16a)}$$

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Referring to Figure 12–2, we see a sharp fillet near this point on the shaft. Then a stress concentration factor of 2.5 should be used:

$$D_6 = \sqrt{\frac{2.94(2.5)(2078)(2)}{25\ 500}} = 1.094 \text{ in}$$

This is very small compared to the other computed diameters, and it will usually be so. In reality, the diameter at D will probably be made much larger than this computed value because of the size of a reasonable bearing to carry the radial load of 2078 lb.

Summary The computed minimum required diameters for the various parts of the shaft in Figure 12–2 are as follows:

$$D_1 = 1.65 \text{ in}$$
$$D_2 = 3.30 \text{ in}$$
$$D_3 = 3.55 \text{ in}$$
$$D_5 = 3.90 \text{ in}$$
$$D_6 = 1.094 \text{ in}$$

Also, D_4 must be somewhat greater than 3.90 in in order to provide adequate shoulders for gear C and bearing B.

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