

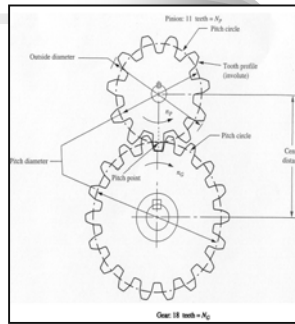
Kinematics of Gears

Chapter 8

Material in this lecture was taken from chapter 8 of Mott, Machine Elements in Mechanical Design, 2003

Kinematics of Gears

- Gears are toothed, cylindrical wheels used for transmitting motion and power from one rotating shaft to another.
- The teeth of a driving gear mesh accurately in the spaces between teeth on the driven gear.



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Speed Reduction Ratio

- Often gears are employed to produce a change in the speed of rotation of the driven gear relative to the driving gear.
- If the smaller top gear (pinion) is driving the larger, lower gear (gear), the larger gear will rotate more slowly.
- The amount of speed reduction is dependent on the ratio of the number of teeth in the pinion to the number of teeth in the gear according to:
 - $n_p / n_g = N_g / N_p$

Kinds of Gears

- The major types of gears are:
 - Spur gears
 - Helical gears
 - Bevel gears
 - Worm/wormgear sets

Kinds of Gears con't

- Spur gears have teeth that are straight and arranged parallel to the axis of the shaft that carries the gear.
- The curved shape of the faces of the spur gear teeth have a special geometry called an involute curve.
- This shape makes it possible for two gears to operate together with smooth, positive transmission of power.

Kinds of Gears con't

- The teeth of helical gears are arranged so that they lie at an angle with respect to the axis of the shaft.
- The angle, called the helix angle, can be virtually any angle.
- Typical helix angles range from approximately 10° to 30°, but angles up to 45° are practical.
- The helical teeth operate more smoothly than equivalent spur gear teeth, and stresses are lower.

Kinds of Gears con't

- Bevel gears have teeth that are arranged as elements on the surface of a cone.
- The teeth of straight bevel gears appear to be similar to spur gear teeth, but they are tapered, becoming wider at the outside and narrower at the top of the cone.
- Bevel gears typically operate on shafts that are 90° to each other.
- Specially designed bevel gears can operate on shafts that are at some angle other than 90°.

Kinds of Gears con't

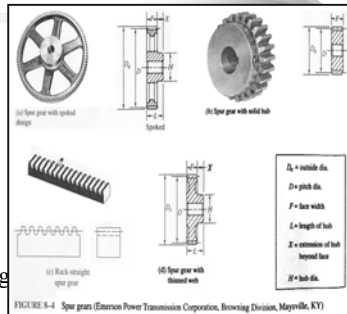
- When both bevel gears in a pair have the same number of teeth, they are called miter gears and are used only to change the axes of the shafts to 90 degrees.
- No speed change occurs.
- A rack is a straight gear that moves linearly instead of rotating.
- When a circular gear is mated with a rack, the combination is called a rack and pinion drive.

Kinds of Gears con't

- A worm and its mating wormgear operate on shafts that are at 90° to each other.
- They typically accomplish a rather large speed reduction ratio compared to other types of gears.
- The worm is the driver, and the wormgear is the driven gear.
- The teeth on the worm appear similar to screw threads, and are called threads rather than teeth.

Spur Gear Styles

- When gears are large, the spoked design in part (a) is often used to save weight.
- The gear teeth are machined into a relatively thin rim that is held by a set of spokes connecting to the hub.



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Spur Gear Styles con't

- A keyway is usually machined into the bore to allow a key to be inserted for positive transmission of torque
- The solid hub design is typical of smaller spur gears.
- When spur gear teeth are machined into a straight, flat bar, the assembly is called a rack.
- The rack is essentially a spur gear with an infinite radius.

Spur Gear Geometry Involute-Tooth Form

- The most widely used spur gear tooth form is the full-depth involute form.
- When two such gear teeth are in mesh and rotating, there is a constant angular velocity ratio between them: From the moment of initial contact to the moment of disengagement, the speed of the driving gear is in a constant proportion to the speed of the driven gear.
- The resulting action of the two gears is very smooth.

Spur Gear Nomenclature

- One of the most important observations that can be made is that throughout the engagement cycle there are two circles, one from each gear, that remain tangent.
- These are called the pitch circles.
- The diameter of the pitch circle of a gear is its pitch diameter; the point of tangency is the pitch point.

Spur Gear Nomenclature con't

- When two gears mesh, the smaller gear is called the pinion, and the larger is the gear.
- The symbol D_p indicates the pitch diameter of the pinion.
- The symbol D_G indicates the pitch diameter of the gear.
- When referring to the number of teeth, use N_p for the pinion and N_G for the gear.

Pitch

- The spacing between adjacent teeth and the size of the teeth are controlled by the pitch of the teeth.
- Three types of pitch designation systems are in common use for gears:
 1. Circular pitch
 2. Diametral pitch
 3. The metric module

Pitch con't

- Circular pitch, p , is the distance from a point on a tooth of a gear at the pitch circle to a corresponding point on the next adjacent tooth, measured along the pitch circle.
- To compute the value of the circular pitch, take the circumference of the pitch circle and divide it into a number of equal parts corresponding to the number of teeth in the gear.

Pitch con't

- The pitch of two gears must be identical.
- $P = \pi D_G / N_G = \pi D_P / N_P$
- The diametral pitch system, P_d , is the most common pitch system used today in the US (the number of teeth per inch of pitch diameter).
- $P_d = N_G / D_G = N_P / D_P$

Gear-tooth Size as Function of Diametral Pitch



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Metric Module System

- The pitch of gears in the metric system is based on this unit and is designated the module, m .
- To find the module of a gear, divide the pitch diameter of the gear in millimeters by the number of teeth.
- $m = D_G / N_G = D_p / N_p$

Backlash

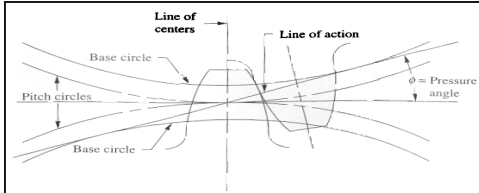
- If the tooth thickness were made identical in value to the tooth space, as it theoretically is, the tooth geometry would have to be absolutely precise for the gears to operate, and there would be no space available for lubrication of the tooth surfaces.
- To alleviate these problems, practical gears are made with the tooth space slightly larger than the tooth thickness, the difference being called the backlash.

Backlash

- To provide backlash, the cutter generating the gear teeth can be fed more deeply into the gear bank than the theoretical value on either or both of the mating gears.
- The magnitude of backlash depends on the desired position of the gear pair and on the size and the pitch of the gears.

Pressure Angle

- The pressure angle is the angle between the tangent to the pitch circles and the line drawn normal (perpendicular) to the surface of the gear teeth.

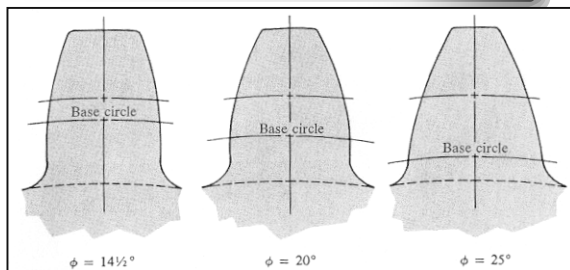


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Pressure Angle con't

- The normal line is sometimes referred to as the line of action.
- When two gear teeth are in mesh and are transmitting power, the force transferred from the driver to the driven gear tooth acts in a direction along the line of action.

Full-Depth, Involute-Tooth Form



Φ is the pressure angle.

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Contact Ratio

- When two gears mesh, it is essential for smooth operation that a second tooth begin to make contact before a given tooth disengages.
- The term contact ratio is used to indicate the average number of teeth in contact during the transmission of power.
- A recommended minimum contact ratio is 1.2 and typical spur gear combinations often have values of 1.5 or higher.

Contact Ratio con't

- The contact ratio is defined as the ratio of the length of the line-of-action to the base pitch for the gear.
- The line-of-action is the straight-line path of a tooth from where it encounters the outside diameter of the mating gear to the point where it leaves engagement.

Contact Ratio con't

- A convenient formula for computing the contact ratio is:

$$m_r = \frac{\sqrt{R_{op}^2 - R_b^2} + \sqrt{R_{om}^2 - R_b^2} - C \sin \phi}{p \cos \phi}$$

- Where ϕ = pressure angle
- R_{op} = outside radius of the pinion
 - $D_{op} / 2 = (N_p + 2) / (2P_d)$
- R_{bp} = radius of the base circle for pinion
 - $D_{bp} / 2 = (D_p / 2) \cos \phi = (N_p / 2P_d) \cos \phi$

Interference Between Mating Spur Teeth

- For certain combinations of numbers of teeth in a gear pair, there is interference between the tip of the teeth on the pinion and the fillet or root of the teeth on the gear.
- The probability that interference will occur is greatest when a small pinion drives a larger gear, with the worst case being a small pinion driving a rack.
- The surest way to prevent this is to control the minimum number of teeth in the pinion to the limiting values shown on the next slide.

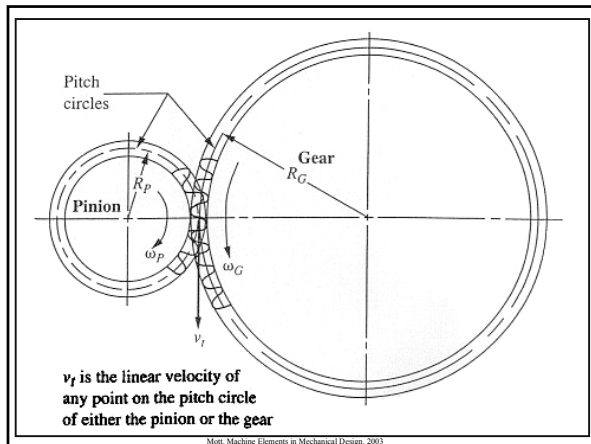
TABLE 8-6 Number of pinion teeth to ensure no interference

Tooth form	For a pinion meshing with a rack		For a 20°, full-depth pinion meshing with a gear	
	Minimum number of teeth		Number of pinion teeth	Maximum number of gear teeth
14½°, involute, full-depth	32		17	1309
20°, involute, full-depth	18		16	101
25°, involute, full-depth	12		15	45
			14	26
			13	16

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Velocity Ratio

- Without slipping there is no relative motion between the two pitch circles at the pitch point, and therefore the linear velocity of a point on either pitch circle is the same.
- Use the symbol v_t for this velocity.
- The linear velocity of a point that is in rotation at a distance R from its center of rotation and rotating with an angular velocity, w , is found from $v_t = R\omega$.



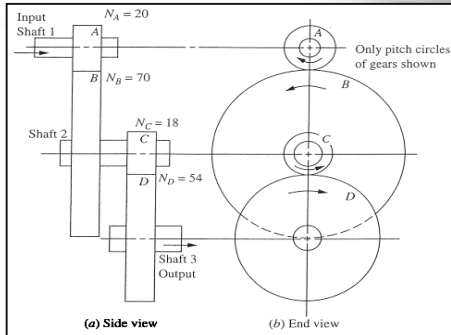
Velocity Ratio con't

- Using the subscript P for the pinion and G for the gear for two gears in mesh:
 - $v_t = R_P \omega_P$
 - $v_t = R_G \omega_G$
- Velocity ratio is defined as:
 - $VR = \omega_P / \omega_G = R_G / R_P$

Train Value

- When more than two gears are in mesh, the term train value (TV) refers to the ratio of the input speed (for the first gear in the train) to the output speed (for the last gear in the train). By definition the train value is the product of the values of VR for each gear pair in the train.
- A gear pair is any set of two gears with a driver and a follower (driven) gear.

Double-Reduction Gear Train



Train Value

- $TV = (N_B / N_A) * (N_D / N_C)$
- $TV = \frac{\text{product of number of teeth in the driven gears}}{\text{product of number of teeth in the driving gears}}$
- This is the form for train value that is used most often

Train Value con't

- The velocity ratios are:
 - $VR_1 = n_A / n_B$
 - $VR_2 = n_C / n_D$
- The train value is:
 - $TV = (VR_1)(VR_2) = (n_A / n_B)(n_C / n_D)$
- But because they are on the same shaft, $n_B = n_C$, and the preceding equation reduces to:
 - $TV = n_A / n_D$

Idler Gear

- Any gear in a gear train that performs as both a driving gear and a driven gear is called an idler gear, or simply an idler.
- The main features of an idler are:
 - An idler does not affect the train value of a gear train because, since it is both a driver and a driven gear, its number of teeth appears in both the numerator and the denominator of the train value equation.

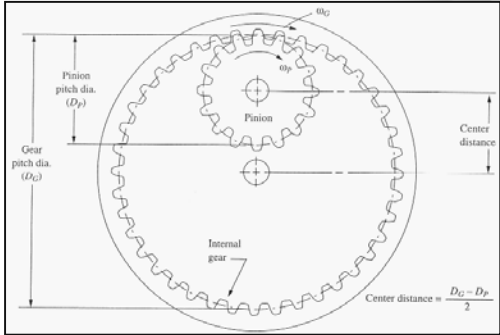
Idler Gear con't

- Placing an idler in a gear train causes a directional reversal of the output gear.
- An idler gear may be used to fill a space between two gears in a gear train when the desired distance between their centers is greater than the center distance for the two gears alone.

Internal Gear

- An internal gear is one for which the teeth are machined on the inside of a ring instead of on the outside of a gear blank.

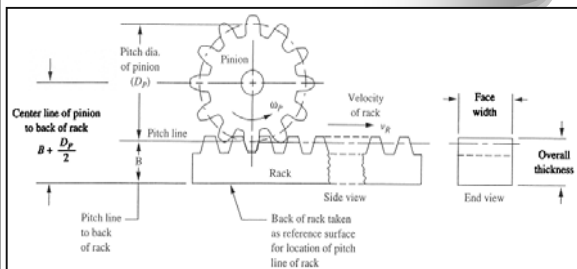
Internal Gear Driven By an External Pinion



Velocity of a Rack

- The function of a rack-and-pinion drive is to produce a linear motion of the rack from the rotational motion of the driving pinion.
- The linear velocity of a rack, v_R , must be the same as the pitch line velocity of the pinion, v_t .
 - $v_R = v_t = R_p \omega_p = (D_p / 2) \omega_p$

Rack Driven by a Pinion



Helical Gear Geometry

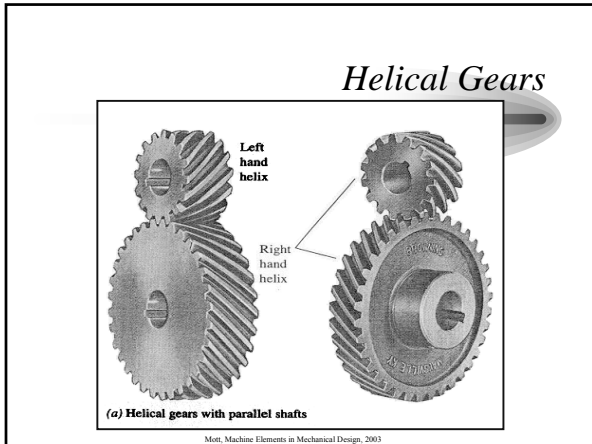
- Helical and spur gears are distinguished by the orientation of their teeth.
- On spur gears, the teeth are straight and are aligned with the axis of the gear.
- On helical gears, the teeth are inclined at an angle with the axis, that angle being called the helix angle.

Helix Angle

- The helix for a given gear can be either left-hand or right-hand.
- The teeth of a right-hand helical gear would appear to lean to the right when the gear is lying on a flat surface and vice versa for the left-hand gear.
- In normal installation, helical gears would be mounted on parallel shafts.
- To achieve this arrangement, it is required that one gear be of the right-hand design and that the other be of the left-hand with an equal helix angle.

Helix Angle

- If both gears in mesh are of the same hand, the shafts will be at 90° to each other.
 - Such gears are called crossed helical gears.
- The parallel shaft arrangement for helical gears is preferred because it results in a much higher power-transmitting capacity for a given size of gear than the crossed helical arrangement.



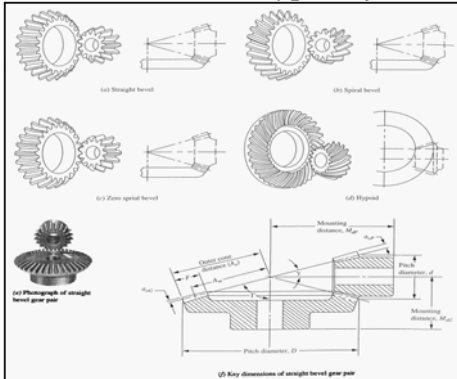
Bevel Gear Geometry

- Bevel gears are used to transfer motion between nonparallel shafts, usually at 90° to one another.
- The 4 primary styles of bevel gears are straight bevel, spiral bevel, zero spiral bevel, and hypoid.
- The surface on which bevel gear teeth are machined is inherently a part of a cone.
- The differences occur in the specific shape of the teeth and in the orientation of the pinion relative to the gear.

Straight Bevel Gears

- The teeth of a straight bevel gear are straight and lie along an element of the conical surface.
- Lines along the face of the teeth through the pitch circle meet at the apex of the pitch cone.
- The centerlines of both the pinion and the gear also meet at this apex.
- In the standard configuration, the teeth are tapered toward the center of the cone.

Types of Bevels



(e) Key dimensions of straight bevel gear pair
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Spiral Bevel Gears

- The teeth of a spiral bevel gear are curved and sloped with respect to the surface of the pitch cone.
- Spiral angles, ψ , of 20° to 45° are used, with 35° being typical.

Zero Spiral Bevel Gears

- The teeth of a zero spiral bevel gear are curved somewhat as in a spiral bevel gear, but the spiral angle is zero.
- These gears can be used in the same mounting as straight bevel gears, but they operate more smoothly.

Hypoid Gears

- The major difference between hypoid gears and the others is offset either above or below the centerline of the gear.
- The teeth are designed specially for each combination of offset distance and spiral angle of the teeth.
- A major advantage is the more compact design that results, particularly when applied to vehicle drive trains and machine tools.

Types of Wormgearing

- Wormgearing is used to transmit motion and power between nonintersecting shafts, usually at 90° to each other.
- The drive consists of a worm on the high-speed shaft which has the general appearance of a power screw thread: a cylindrical, helical thread.
- The worm drives a wormgear, which has an appearance similar to that of a helical gear.

Types of Wormgearing con't

- Worms and wormgears can be provided with either right-hand or left-hand threads on the worm.
- Several variations of the geometry of wormgear drives are also available.
- The most common one employs a cylindrical worm mating with a wormgear having teeth that are throated, wrapping partially around the worm.
 - This is called a single-enveloping type of wormgear.

Worms and Wormgears

