Designs for Different Types of Loading

Chapter 5

Material taken from Chapter 5 Mott, 2003, Machine Elements in Mechanical Design

Loading Types

- The manner of computing the design stress depends on the manner of loading and on the type of material. Loading types include the following:
  - Static
  - Repeated and reversed
  - Fluctuating
  - Shock or impact
  - Random

Types of Loading and Stress Ratio

- The primary factors to consider when specifying the type of loading to which a machine part is subjected are the manner of variation of the load and the resulting variation of stress with time.
- Stress variations are characterized by four key values:
  - Maximum stress, σ_{max}
  - Minimum stress, σ_{min}
  - Mean (average) stress, σ_m
  - Alternating stress, σ_a (stress amplitude)

Types of Loading and Stress Ratio

- The maximum and minimum stresses are usually computed from known information by stress analysis or finite-element methods, or they are measured using experimental stress analysis techniques.
- Then the mean and alternating stresses can be computed from:
  - $\sigma_m = (\sigma_{max} + \sigma_{min}) / 2$
  - $\sigma_a = (\sigma_{max} - \sigma_{min}) / 2$

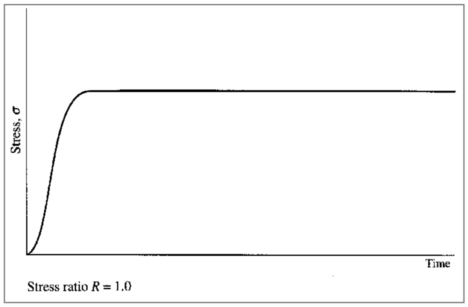
Types of Loading and Stress Ratio

- The behavior of a material under varying stresses is dependent on the manner of the variation. One method used to characterize the variation is called stress ratio.
- Stress ratio $R = \frac{\text{minimum stress}}{\text{maximum stress}} = \frac{\sigma_{min}}{\sigma_{max}}$
- Stress ratio $A = \frac{\text{alternating stress}}{\text{mean stress}} = \frac{\sigma_a}{\sigma_m}$

Static Stress

- When a part is subjected to a load that is applied slowly, without shock, and is held at a constant value, the resulting stress in the part is called static stress.
- Because $\sigma_{max} = \sigma_{min}$, the stress ratio for static stress is R = 1.0.

Static Stress



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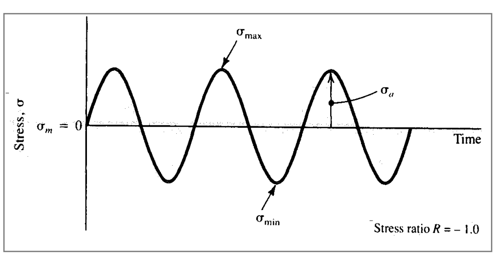
Repeated and Reversed Stress

- A stress reversal occurs when a given element of a load-carrying member is subjected to a certain level of tensile stress followed by the same level of compressive stress.
- The stress is called repeated and reversed.
- Because $\sigma_{min} = -\sigma_{max}$, the stress ratio is R = -1.0, and the mean stress is zero.

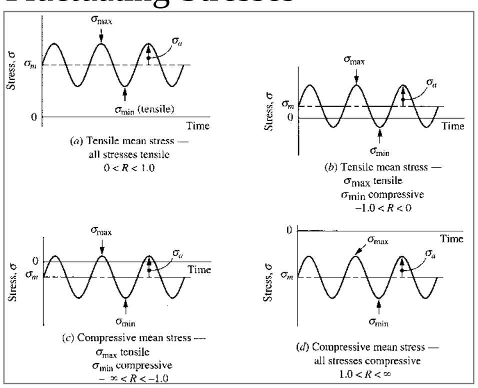
Repeated, Reversed Stress



Fluctuating Stress

- When a load-carrying member is subjected to an alternating stress with a nonzero mean, the loading produces fluctuating stress.

Fluctuating Stresses



(a) Tensile mean stress — all stresses tensile $0 < R < 1.0$

(b) Tensile mean stress — σ_{max} tensile σ_{min} compressive $-1.0 < R < 0$

(c) Compressive mean stress — σ_{max} tensile σ_{min} compressive $-\infty < R < -1.0$

(d) Compressive mean stress — all stresses compressive $1.0 < R < \infty$

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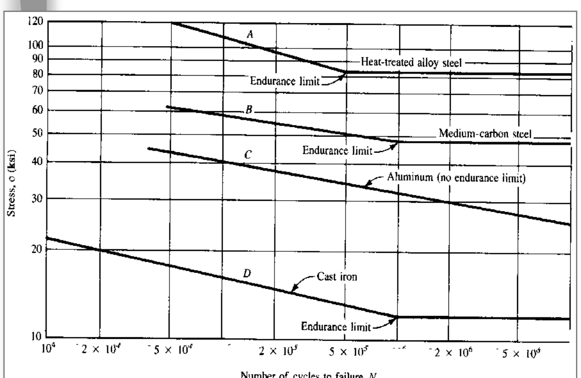
Shock, Impact, and Random Loading

- Loads applied suddenly and rapidly cause shock or impact.
- Examples include a hammer blow, a weight falling onto a structure, and the action inside a rock crusher.
- When varying loads are applied that are not regular in their amplitude, the loading is called random.

Endurance Strength

- The endurance strength of a material is its ability to withstand fatigue loads. In general, it is the stress level that a material can survive for a given number of cycles of loading. If the number of cycles is infinite the stress level is called the endurance limit.
- Endurance strengths are usually charted on a graph called an S-N diagram.
- Curves A, B, and D show a material that exhibits an endurance limit, such as plain carbon steel. Curve C is typical of most nonferrous materials, such as aluminum, which does not have an endurance limit.



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Endurance Strength

- Approximations for the basic endurance strength for wrought steel:
  - Endurance strength = 0.50(ultimate tensile strength) = 0.50(s_u)

Estimated Actual Endurance Strength

- If the actual material characteristics or operating conditions for a machine part are different from those for which the basic endurance strength was determined, the fatigue strength must be reduced from the reported value.

1. Specify the material for the part and determine its ultimate tensile strength, s_u, considering its condition, as it will be used in service.
2. Specify the manufacturing process used to produce the part with special attention to the condition of the surface in the most highly stressed area.
3. Use Figure 5–8 to estimate the modified endurance strength, s_n.
4. Apply a material factor, C_m, from the following list.

Wrought steel:	$C_m = 1.00$	Malleable cast iron:	$C_m = 0.80$
Cast steel:	$C_m = 0.80$	Gray cast iron:	$C_m = 0.70$
Powdered steel:	$C_m = 0.76$	Ductile cast iron:	$C_m = 0.66$

5. Apply a type-of-stress factor: $C_{st} = 1.0$ for bending stress; $C_{st} = 0.80$ for axial tension.
6. Apply a reliability factor, C_R, from Table 5–1.
7. Apply a size factor, C_s, using Figure 5–9 and Table 5–2 as guides.
8. Compute the estimated actual endurance strength, s_n', from

$$s_n' = s_n (C_m)(C_{st})(C_R)(C_s) \quad \textbf{(5–4)}$$

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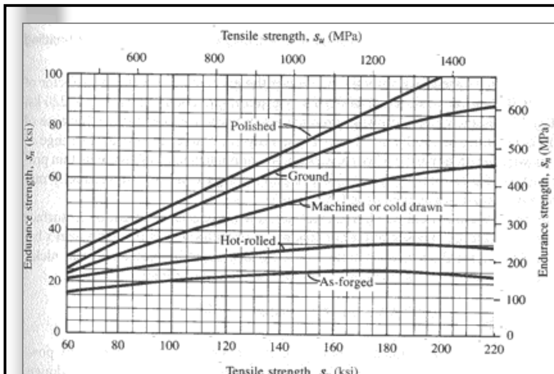

FIGURE 5–8 Endurance strength s_n versus tensile strength for wrought steel for various surface conditions

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Approximate Reliability Factors

Desired reliability	C_R
0.50	1.0
0.90	0.90
0.99	0.81
0.999	0.75

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Size Factor

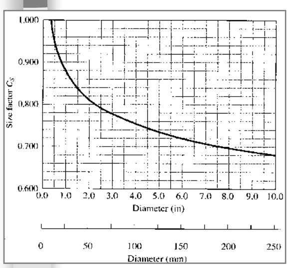

TABLE 5–2 Size factors

U.S. customary units	
Size Range	For D in inches
$D \le 0.30$	$C_S = 1.0$
$0.30 < D \le 2.0$	$C_S = (D/0.3)^{-0.11}$
$2.0 < D < 10.0$	$C_S = 0.859\text{-}0.02125D$
SI units	
Size Range	For D in mm
$D \le 7.62$	$C_S = 1.0$
$7.62 < D \le 50$	$C_S = (D/7.62)^{-0.11}$
$50 < D < 250$	$C_S = 0.859\text{-}0.000837D$

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Example Problem 5–2 Estimate the actual endurance strength of AISI 1050 cold-drawn steel when used in a circular shaft subjected to rotating bending only. The shaft will be machined to a diameter of approximately 1.75 in.

Solution

Objective: Compute the estimated actual endurance strength of the shaft material.

Given:
AISI 1050 cold-drawn steel, machined.
Size of section: $D = 1.75$ in.
Type of stress: Reversed, repeated bending.

Analysis: Use the Procedure for Estimating Actual Endurance Strength, s_n'

Step 1: The ultimate tensile strength: $s_u = 100$ ksi from Appendix 3.
Step 2: Diameter is machined.
Step 3: From Figure 5–8, $s_n = 38$ ksi
Step 4: Material factor for wrought steel: $C_m = 1.0$
Step 5: Type-of-stress factor for reversed bending: $C_{st} = 1.0$
Step 6: Specify a desired reliability of 0.99. Then $C_R = 0.81$ (Design decision)
Step 7: Size factor for circular section with $D = 1.75$ in.
From Figure 5–9, $C_s - 0.83$.
Step 8: Use Equation 5–4 to compute the estimated actual endurance strength.

$$s_n' = s_n(C_m)(C_{st})(C_R)(C_s) = 38\text{ ksi}(1.0)(1.0)(0.81)(0.83) = 25.5\text{ ksi}$$

Comments: This is the level of stress that would be expected to produce fatigue failure in a rotating shaft due to the action of reversed bending. It accounts for the basic endurance strength of the wrought AISI 1050 cold-drawn material, the effect of the machined surface, the size of the section, and the desired reliability.

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Design Philosophy

- It is the designer's responsibility to ensure that a machine part is safe for operation under reasonably foreseeable conditions.
- Some general conditions are:
  - Application: quantities, manufacturing techniques, danger to people and economic cost, cost-sensitive, small physical size or low weight
  - Environment: temperature range, electrical voltage or current, low noise, vibration environment
  - Loads: consider all modes of operation, including startup, shut down, normal operation, and foreseeable overloads. The loads should be characterized as static, repeated and reversed, fluctuation, shock, or impact.

Design Philosophy con't

- Types of Stresses: direct tension, direct compression, direct shear, bending, or torsional
- Material: material properties of yield strength, ultimate tensile strength, ultimate compressive strength, endurance strength, stiffness, ductility, toughness, creep resistance, and corrosion resistance
- Confidence: reliable data for loads, material properties, and stress calculations.

Design Philosophy con't

- All design approaches must define the relationship between the applied stresses on a component and the strength of the material from which it is being made, considering the conditions of service.
- The goal of the design process is to achieve a suitable design factor, N (aka factor of safety), that ensures the component is safe. The strength of the material must be greater than the applied stresses.

Design Philosophy con’t

- The sequence of design analysis is:
  - Geometry of the component and the loading are known: apply the desired design factor, N, to the actual expected stress to determine the required strength of the material. Then a suitable material can be specified.
  - Loading is known and the material for the component has been specified: compute a design stress by applying the desired design factor, N, to the approximate strength of the material. This is the maximum allowable stress to which any part of the component can be exposed. Then complete the stress analysis to determine what shape and size of the component will ensure that stresses are safe.

Design Philosophy con’t

  - Loading is known, and the material and the complete geometry of the component have been specified: compute both the expected maximum applied stress and the design stress. By comparing these stresses, determine the resulting design factor, N, for the proposed design and judge its acceptability. A redesign may be called for if the design factor is either too low (unsafe) or too high (over designed).

Design Philosophy con’t

- Machine elements can fail because of excessive deformation or vibration.
- Criteria for failure due to deformation are often highly dependent on the machine’s use.
  - Will excessive deformation cause two or more members to touch when they should not?
  - Will the desired precision of the machine be compromised?
  - Will parts vibrate excessively or resonate at the frequencies experienced during operation?

Design Factors

- The term design factor, N, is a measure of a load-carrying component safety. The strength of the material from which the component is to be made is divided by the design factor to determine a design stress, σ_d (aka allowable stress).Then the actual stress to which the component is subjected should be less than the design stress.

Design Factors con't

- Often the value of the design factor or the design stress is governed by codes established by standards-setting organizations such as the American Society of Mechanical Engineers, the American Gear Manufacturers Associations, or the US Dept of Defense.
- In the absence of codes or standards, the designer must use judgment to specify the desired design factor.

Ductile Materials

1. N = 1.25 to 2.0. Design of structures under static loads for which there is a high level of confidence in all design data.
2. N = 2.0 to 2.5. Design of machine elements under dynamic loading with average confidence in all design data. (typically used in problem solutions)
3. N = 2.5 to 4.0. Design of static structures or machine elements under dynamic loading with uncertainty about loads, material properties, stress analysis, or the environment.

Ductile Materials con’t

4. N = 4.0 or higher. Design of static structures or machine elements under dynamic loading with uncertainty about some combination of loads, material properties, stress analysis, or the environment. The desire to provide extra safety to critical components may also justify these values.

Brittle Materials

5. N = 3.0 to 4.0. Design of structures under static loads for which there is a high level of confidence in all design data.
6. N = 4.0 to 8.0. Design of static structures or machine elements under dynamic loading with uncertainty about loads, material properties, stress analysis, or the environment.

Predictions of Failure

- Designers should understand the various ways that load-carrying components can fail in order to complete a design that ensures that failure does not occur. Several different methods of predicting failure are available, and it is the designer’s responsibility to select the one most appropriate to the conditions of the project.

Failure Prediction Method

1. Maximum normal stress
   - Uniaxial static stress on brittle materials
2. Modified Mohr
   - Biaxial static stress on brittle materials
3. Yield Strength
   - Uniaxial static stress on ductile materials
4. Maximum Shear Stress
   - Biaxial static stress on ductile materials [moderately conservative]
5. Distortion Energy
   - Biaxial or triaxial on ductile materials [good predictor]

Failure Prediction Method

6. Goodman
   - Fluctuating stress on ductile materials [slightly conservative]
7. Gerber
   - Fluctuation stress on ductile materials [good predictor]
8. Soderberg
   - Fluctuating stress on ductile materials [moderately conservative]

Selected Review of Failure Prediction Methods

Maximum Normal Stress Method

- The maximum normal stress theory states that a material will fracture when the maximum normal stress (either tension or compression) exceeds the ultimate strength of the material as obtained from a standard tensile or compressive test. Its use is limited, namely for brittle materials under pure uniaxial static tension or compression. When applying this theory, any stress concentration factor at the region of interest should be applied to the computed stress because brittle materials do not yield and cannot redistribute the increased stress.

Maximum Normal Stress Theory

- For tensile stress:
  - $K_t\sigma < \sigma_d = s_{ut} / N$
- For compressive stress:
  - $K_t\sigma < \sigma_d = s_{uc} / N$

Where K_t is the stress concentration factor and σ is the calculated stress

Yield Strength Method

- This is a simple application of the principle of yielding in which a component is carrying a direct tensile or compressive load in the manner similar to the conditions of the standard tensile or compressive test for the material. Failure is predicted when the actual applied stress exceeds the yield strength. Stress concentrations can normally be neglected for static stresses on ductile materials because the higher stresses near the stress concentrations are highly localized.

Yield Strength Method con't

- For tensile stress:
  - $\sigma < \sigma_d = s_{yt} / N$
- For compressive stress:
  - $\sigma < \sigma_d = s_{yc} / N$
- For most wrought ductile metals:
  - $s_{yt} = s_{yc}$

Maximum Shear Stress Method

- The maximum shear stress method of failure prediction states that a ductile material begins to yield when the maximum shear stress in a load-carrying component exceeds that in a tensile-test specimen when yielding begins. A Mohr's circle analysis for the uniaxial tension test shows that the maximum shear stress is ½ of the applied tensile stress. At yield then, $s_{sy} = s_y / 2$
- $\tau_{max} < \tau_d = s_{sy} / N = 0.5\ s_y / N$
- The maximum shear stress method of failure prediction has always shown by experimentation to be somewhat conservative for ductile materials subjected to a combination of normal and shear stresses.

Distortion Energy Method

- The distortion energy method has been shown to be the best predictor of failure for ductile materials under static loads or completely reversed normal, shear, or combined stresses. It requires the definition of von Mises stress, indicated by the symbol σ', that can be calculated for biaxial stresses, given the maximum and minimum principle stresses, σ_1 and σ_2 from:

$$\sigma' = \sqrt{\sigma_1^2 + \sigma_2^2 - \sigma_1\sigma_2}$$

Distortion Energy Method con't

- Failure is predicted when $\sigma' > s_y$. The biaxial stress approach requires that the applied stress in the third orthogonal direction, typically σ_z, is zero.
- The failure line is an ellipse centered at the origin and passing through the yield strength on each axis, in both the tensile and compressive regions. It is necessary that the material has equal values for yield strength in tension and compression for direct use of this method.

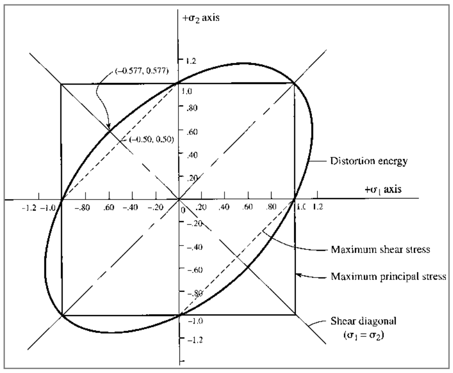
Distortion Energy Method con't

- The numerical scales on the graph are normalized to the yield strength so the ellipse passes through $s_y/\sigma_1 = 1$ on the σ_1 axis and similarly on the other axes.
- Combinations of principle stresses that lie within the distortion energy ellipse are predicted to be safe, while those outside would predict failure.

Distortion Energy Method con't

- For design, the design factor, N, can be applied to the yield strength.
- $\sigma' < \sigma_d = s_y / N$

Distortion Energy Method



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Alternate Form for the von Mises Stress

- Often the stresses are found in a somewhat orthogonal direction, x and y, namely σ_x, σ_y, and τ_{xy}. The von Mises stress can then be calculated directly from $\sigma' = \sqrt{\sigma_x^2 + \sigma_y^2 - \sigma_x\sigma_y + 3\tau_{xy}^2}$
- For uniaxial stress with shear, $\sigma_y = 0$…

$$\sigma' = \sqrt{\sigma_x^2 + 3\tau_{xy}^2}$$

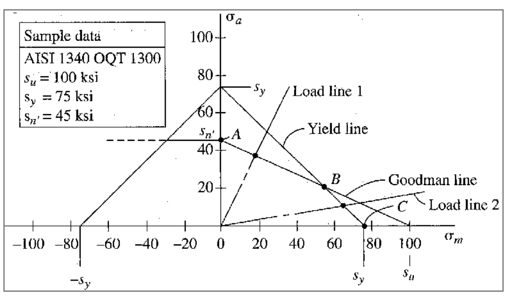
Goodman Method

- The Goodman method of failure prediction provides a good correlation with experimental data.
- The Goodman diagram plots the mean stresses on the horizontal stress and the alternating stresses on the vertical axis.
- A straight line is drawn from the estimated actual endurance strength of the material, s'_n, on the vertical axis to the ultimate tensile strength, s_u, on the horizontal axis.
- Combinations of mean stress, σ_m, and alternating stress, σ_a, above the line predict failure, while those below the line predict no failure.

Goodman Method con't

$$\frac{\sigma_a}{s'_n} + \frac{\sigma_m}{s_u} = 1$$

Modified Goodman Diagram



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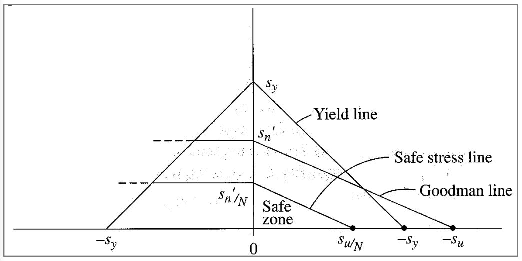
Design Equation

- Introducing a design factor on both the ultimate and endurance strength values depicts a "safe stress" line.
- Any stress concentration factor in the region of interest should be applied to the alternating component but not to the mean stress component, because experimental evidence shows that the presence of a stress condition does not affect the contribution of the mean stress of fatigue failure.
- $$\frac{K_t \sigma_a}{s'_n} + \frac{\sigma_m}{s_u} = \frac{1}{N}$$

Modified Goodman Diagram



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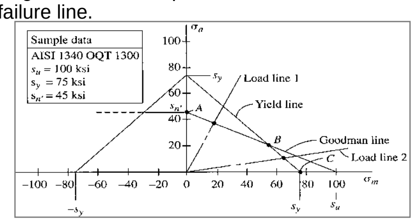
Early Cycle Yielding

- The Goodman line presents a difficulty near the right end because it seems to allow a pure mean stress greater than the yield strength of the material.
- Most designers choose to not permit yielding anywhere. The yield line is added to the Goodman diagram, drawn between the yield strength plotted on both axes. Now the line segments between points labeled A, B, and C define the failure line.



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Early Cycle Yielding con’t

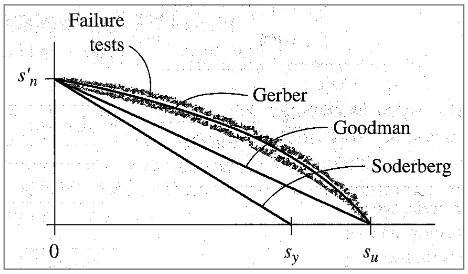
- First complete the design based on fatigue and then check for yield separately.
- The design equation for the yield line:

$$\frac{K_t \sigma_a}{S'_n} + \frac{K_t \sigma_m}{S_y} = 1$$

- Apply the stress concentration factor to the mean stress to ensure that yielding does not occur. In many cases, the safe stress line for fatigue actually falls completely below the yield strength line, indicating that no yielding is expected.

Comparison of 3 Methods

Soderberg is the most conservative.



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Soderberg Method

- $\frac{K_t \sigma_a}{S'_n} + \frac{\sigma_m}{S_y} = 1$
- Drawn between the endurance strength and the yield strength, the Soderberg line is the most conservative. One advantage is that it protects directly against early cycle yielding, whereas Goodman requires the secondary consideration of the yield line. However, the degree of conservatism is considered too great for competitive efficient design.

Design Analysis

- Summarize the recommended methods for design analysis based on the type of material (brittle or ductile), the naure of the loading (static or cyclical), and the type of stress (uniaxial or biaxial).

Design Analysis

- The following symbols are used:
  - s_u or s_{ut} = ultimate tensile strength
  - s_{uc} = ultimate compressive strength
  - s_y = yield strength or yield point
  - s_{sy} = yield strength in shear
  - s'_n = endurance strength of material under actual conditions
  - s'_{sn} = endurance strength in shear under actual conditions
  - σ = nominal applied stress, without Kt

General Design Procedure

- The procedure is set up assuming that the following factors are known or can be estimated:
  - General design requirements: objectives and limitations on size, shape, weight, desired precision, etc
  - Nature of the loads to be carried
  - Types of stresses produced by the loads
  - Type of material from which the element is to be made
  - General description of the manufacturing process to be used, particularly with regard to the surface finish that will be produced
  - Desired reliability

General Design Procedure

1. Specify the objectives and limitations, if any, of the design, including desired life, size, shape, and appearance.
2. Determine the environment in which the element will be placed, considering such factors as corrosion potential and temperature
3. Determine the nature and characteristics of the loads to be carried by the element
4. Determine the magnitudes for the loads and the operating conditions

General Design Procedure con't

5. Analyze how loads are to be applied to determine the type of stresses produced
6. Propose the basic geometry for the element
7. Propose the method of manufacturing the element
8. Specify the material from which the element is to be made, along with its condition
9. Determine the expected properties of the selected material
10. Specify an appropriate design factor
11. Determine stress analysis method

General Design Procedure con't

12. Compute the appropriate design stress for use in the stress analysis
13. Determine the nature of any stress concentrations
14. Complete the required stress analyses at all points where the stress may be high
15. Specify suitable, convenient dimensions for all features of the element

General Design Procedure con't

16. Check all assumptions made earlier in the design to ensure that the element is still safe and reasonably efficient.
17. Specify suitable tolerances for all dimensions
18. Check to determine whether some part of the component may deflect excessively
19. Document the final design with drawings and specifications
20. Maintain a careful record of the design analyses for future reference.