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An Improved Green–Ampt Infiltration and Redistribution Method for Uneven Multistorm Series

L. Gowdish and R. Muñoz-Carpena*

Robust hydrologic models require an accurate formulation of infiltration and soil water redistribution. The application of Richards' equation can provide the most accurate description of these processes but for some applications it can be computationally intensive and prone to numerical instability and convergence errors. A conceptual, physically based formulation like the Green–Ampt with Redistribution (GAR) can be an attractive alternative in many applications. Original GAR applications, however, showed significant errors in simulated surface water content for soils with high saturated hydraulic conductivity values, and an increasing surface water content deviation after subsequent redistributions during long simulations. A modified GAR method (MGAR) is proposed that provides improved infiltration and soil water redistribution predictions during uneven multistorm time series for a wide range of soils. An increased number of redistributing wetting fronts more accurately represents the naturally curvilinear soil water content profile during the redistribution phase. A redistribution coefficient decreases the surface soil water prediction during nonuniform precipitation series as a function of three variables: saturated hydraulic conductivity, redistribution number, and redistribution time for each storm event in the time series. Simulations of uneven multistorm precipitation time series using GAR and MGAR for 11 soil textural classifications were compared against Richards' equation. The MGAR markedly improved surface soil water predictions (coefficients of efficiency >0.935 and RMSE <0.011). The method also provided a good approximation of average water content for soil observation depths within the top 1 m, corresponding with the area of interest for many vadose zone modeling applications.

Abbreviations: GAR, Green–Ampt with Redistribution; MGAR, modified Green–Ampt with Redistribution; WF, wetting front.

Vertical unsaturated flow can best be described by combining Darcy's law with the continuity equation, resulting in a partial differential equation known as Richards' equation (Richards, 1931). Richards' equation does not have a general analytical solution and therefore must be solved numerically in many practical applications. The solutions of Richards' equation can be computationally intensive, requiring extensive soil property data and involving parameterization and fine spatial and temporal discretization, which can result in errors (Skaggs and Khaleel, 1982; Ogden and Saghafian, 1997). In addition, certain conditions (e.g., coarse soil types and highly dynamic boundary conditions) can present problems of calibration, instability, and errors of convergence that compromise the solutions (Celia et al., 1990; Paniconi and Putti, 1994; Miller et al., 1998; Vogel et al., 2001; Seibert 2003). For this reason, approximate, physically based approaches have often been used for modeling infiltration and soil water redistribution (Jury and Horton, 2004; Singh and Woolhiser, 2002; Haan et al., 1993; Smith et al., 1993). Particularly, for the case of infiltration, the method of Green and Ampt (1911), modified for unsteady rain events (Mein and Larson, 1973; Chu, 1978), has been widely used in hydrologic modeling. Despite Green–Ampt's apparent limitations (assumptions of rectangular saturated piston flow and homogeneous isotropic soil with uniform initial content), the method produces good results in comparison with other approximate and numerical methods if it is effectively parameterized (Skaggs et al., 1969). In addition, Green–Ampt has the advantage that its parameters can be estimated directly from soil textural classification (Rawls et al., 1982, 1983).

As opposed to the complete Richards' (1931) formulation, the Green–Ampt equation strictly handles infiltration. Thus, an equation for the redistribution of soil water in the profile between rainfall events is also needed. For example, Groves (1989) proposed a soil water model based on two components for rainfall and dry periods: the Smith and Parlange approach for infiltration and the sharp piston wetting-front formulation of Green–Ampt for redistribution. The USDA-ARS Root Zone Water Quality Model (RZWQM) uses the Green–Ampt equation to calculate infiltration rates into homogeneous or layered soil profiles, while the soil water is redistributed by a numerical solution of the Richards equation (Cameira et al., 2000). Other soil water redistribution models have been proposed and used (Smith et al., 1993; Philip and Knight, 1991; Gardner et al., 1970; Jury et al., 1976), but it is desirable that the redistribution model chosen be based on the same underlying assumptions as the infiltration model. This can result in a combined model with a reduced parameter set,

Agricultural and Biological Engineering Dep., Univ. of Florida. P.O. Box 110570, Gainesville, FL 32611. Received 29 Feb. 2008. *Corresponding author (carpena@ufl.edu).

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which, compared with overparameterized models resulting from the merging of infiltration and redistribution components with different conceptual bases, stands a better chance for its behavior to be tested against measured data (Kirchner, 2006).

For the case of Green–Ampt infiltration, Ogden and Saghafian (1997) described the Green–Ampt with Redistribution (GAR) method. This model is a specific case of the conceptual model developed originally by Smith et al. (1993) and Corradini et al. (1997) to simulate the continuous infiltration and soil water redistribution cycle for multistorm time series using an extension of the Parlange et al. (1982) infiltration equation. These formulations were shown to perform satisfactorily when compared with a numerical solution of the Richards equation (Smith et al., 1993; Corradini et al., 1997; Ogden and Saghafian, 1997) and laboratory data (Melone et al., 2006). In addition, these methods are explicit (i.e., easy to implement) and robust when time steps are constrained to allow convergence. One benefit of using the GAR formulation is that it requires only four soil physical parameters and two soil water contents (saturated and residual), which can be approximated from soil textural classifications (Rawls et al., 1982, 1983).

There are, however, some limitations to the GAR method. Ogden and Saghafian (1997) showed that for soils with relatively large saturated hydraulic conductivity values, the error in the prediction of surface water content increases. Furthermore, when the method is applied to a long period of uneven storms, there is an increasing divergence from actual surface water content after subsequent redistributions.

The purpose of this study was to develop and evaluate modifications to the conceptual GAR that would reduce the error in surface water content predictions for a wide range of soil types during long series of uneven rain and redistribution events. In addition, the capability of the method to predict average soil water within the soil profile was studied. The motivation for this work came from the desire to provide an improved, physically based method for modeling infiltration and soil water redistribution that could be translated to a variety of hydrologic modeling applications.

Infiltration and Redistribution Calculations

Green–Ampt Infiltration with Redistribution Method

The Green and Ampt (1911) equation describes the infiltration process under ponded conditions. Since the surface is not always under ponded conditions, Mein and Larson (1973) modified the equation to determine the time when surface ponding begins under steady rainfall conditions. Chu (1978) further extended the Green–Ampt equation to allow modeling of infiltration under unsteady rainfall. Finally, Skaggs and Khaleel (1982) added mass balance at the soil surface, needed for the calculation of excess rainfall.

The GAR method considers that the rectangular wetting front will elongate uniformly during redistribution due to unsaturated flow caused by capillary and gravitational forces (Charbenau and Asgian, 1991; Fig. 1). To operationally separate the infiltration and redistribution phases, the method assumes that soil water only redistributes during a rainfall hiatus, defined as the period when the rainfall rate is less than the saturated hydraulic conductivity ($r < K_s$) and all ponded surface water has been infiltrated (Smith et al., 1993).

After initial ponding during a rainfall event, the water content at the surface, θ_o, is equal to the saturated water content, θ_s. Once redistribution occurs, the water content at the surface becomes $<\theta_s$. During the rainfall hiatus, the equation of continuity that describes the redistribution of soil water content (Fig. 1) can be represented in differential form as (Smith et al., 1993)

$$\frac{\mathrm{d}}{\mathrm{d}t}\theta_o = \frac{1}{Z}\left\{r - K_i - \left[K\left(\theta_o\right) + \frac{K_s G\left(\theta_i, \theta_o\right)}{Z}\right]\right\} \quad [1]$$

where t [T] is the simulation time, Z [L] is the depth to the wetting front, r [L T^{-1}] is the rainfall rate during the hiatus, K_i [L T^{-1}] is the value of the unsaturated hydraulic conductivity function $K(\theta)$ for the initial soil water content (θ_i), K_s [L T^{-1}] is the saturated hydraulic conductivity of the soil, and $G(\theta_i,\theta_o)$ [L] is the integral of the capillary drive through the saturated front. Equation [1] was solved using a fourth-order Runge–Kutta adaptive step size solution with a tolerance of 1.0×10^{-4} (Press et al., 1989).

Several analytical expressions for the G function exist, depending on the form of the soil water retention and unsaturated conductivity functions chosen (Smith et al., 2002). For Brooks and Corey (1964),

$$\Theta = \frac{\theta - \theta_r}{\theta_s - \theta_r} = \left(\frac{h_b}{h}\right)^{\lambda}$$

$$K_r = \frac{K\left(h\right)}{K_s} = \Theta^{3+2/\lambda} \quad [2]$$

where Θ (dimensionless) is the relative volumetric water content of the soil profile for a generic water content (θ) and its suction (h, [L]); θ_r is the residual water content [L L^{-3}]; h_b and λ are the parameters of the Brooks and Corey (1964) equation, where h_b [L] is the bubbling pressure and λ (dimensionless) is the pore-size distribution index; and K_r is the relative unsaturated hydraulic

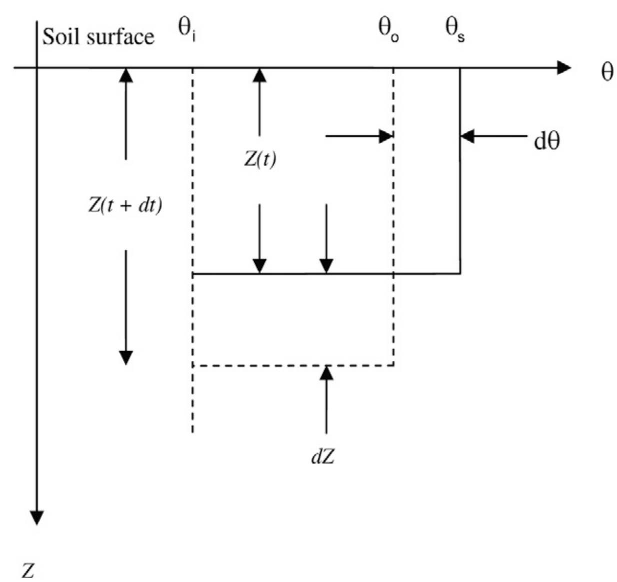

FIG. 1. Conceptual model for soil water profile elongation during redistribution (Ogden and Saghafian, 1997); θ is volumetric water content, Z is the wetting front depth, and t is simulation time.

conductivity function. In this case, Ogden and Saghafian (1997) showed that the function $G(\theta_i,\theta_o)$ can be obtained by

$$G(\theta_i,\theta_o) = S_{av}\left[\frac{(\Theta_o)^{3+1/\lambda} - (\Theta_i)^{3+1/\lambda}}{1-(\Theta_i)^{3+1/\lambda}}\right] \quad [3]$$

$$\text{with } S_{av} = -h_b\frac{2+3\lambda}{1+3\lambda}$$

where S_{av} [L] is the Green–Ampt average suction at the wetting front, Θ_o is the relative volumetric water content of the soil at the surface, and Θ_i is the initial relative volumetric water content of the soil profile. The solution of Eq. [1] and [3] permits calculation of the redistribution of the soil water content during the rainfall hiatus. Since this is a continuation of the Green–Ampt equation and follows the same assumptions, the water content calculated from Eq. [1] is not only the soil surface water content, θ_o, but also the water content of the soil above the wetting front, a distance Z from the surface. This is based on the assumption that water is infiltrating as a wetting front as shown in Fig. 1. Figure 2a represents the process of redistribution during the rainfall hiatus, where the profile of soil water elongates to a depth $Z(t)$ at time t, and its soil water content decreases from saturation, $\Theta = 1$, to $\Theta_o(t)$. Soil water content is allowed to continue to redistribute until a minimum value of soil water content, selected by the user, is reached (e.g., field capacity, FC, or the wilting point).

To better represent the real soil water profile, if during redistribution the rainfall rate increases to $r > K_s$, GAR introduces a second rectangular profile (Wetting Front 2 or WF2) while the original wetting front, WF1, with a depth of Z_1 remains invariant (Fig. 2b). New infiltration will be allocated only to the second wetting front (depth of Z_2 in Fig. 2b) until the two fronts are at the same depth ($Z_1 = Z_2$), at which time they will merge into one profile at saturated water content. It should be noted that the soil water profile formed by the two rectangles better resembles the description of the numerical solution of Richards' equation where the soil profile is represented by two curvatures (Fig. 2b). The Green–Ampt equations for unsteady rainfall are used to calculate the infiltration for the second wetting front and also after the two fronts merge, $Z_1 = Z_2$. If, instead, the rainfall rate drops below the saturated hydraulic conductivity ($r < K_s$) before $Z_1 = Z_2$, the two profiles are forced to merge into one profile with a new depth Z_m calculated as the weighted average of both wetting fronts with respect to their respective soil water contents. Once the two profiles are merged into one profile with depth Z_m, the original Green–Ampt infiltration equations are used again. Ogden and Saghafian (1997) proposed to limit the number of profiles to two to simplify the calculation. They stated, however, that although the union of the two fronts is not physically correct, this simplification introduces a small error in the calculation so long as $(\theta_1 - \theta_i) > (\theta_s - \theta_1)$, where θ_1 is the water content of WF1. This implies that the scheme to merge the two wetting fronts is only valid for relatively short redistribution intervals.

Modified Green-Ampt Infiltration with Redistribution Method

Multiple Redistributing Wetting Front Scheme

Although limiting the number of wetting fronts to two simplifies the calculation, it limits the physical basis in comparison to numerical and experimental soil water profiles. When the two fronts merge, a portion of the soil profile that was at θ_1, where $\theta_i > \theta_1 > \theta_s$, is now instantaneously changed to saturation and another portion changed to θ_i, which can lead to discontinuous "jumps" in water content along these soil sections. This scenario is physically unrealistic. In addition, as explained above, the condition $(\theta_1 - \theta_i)$

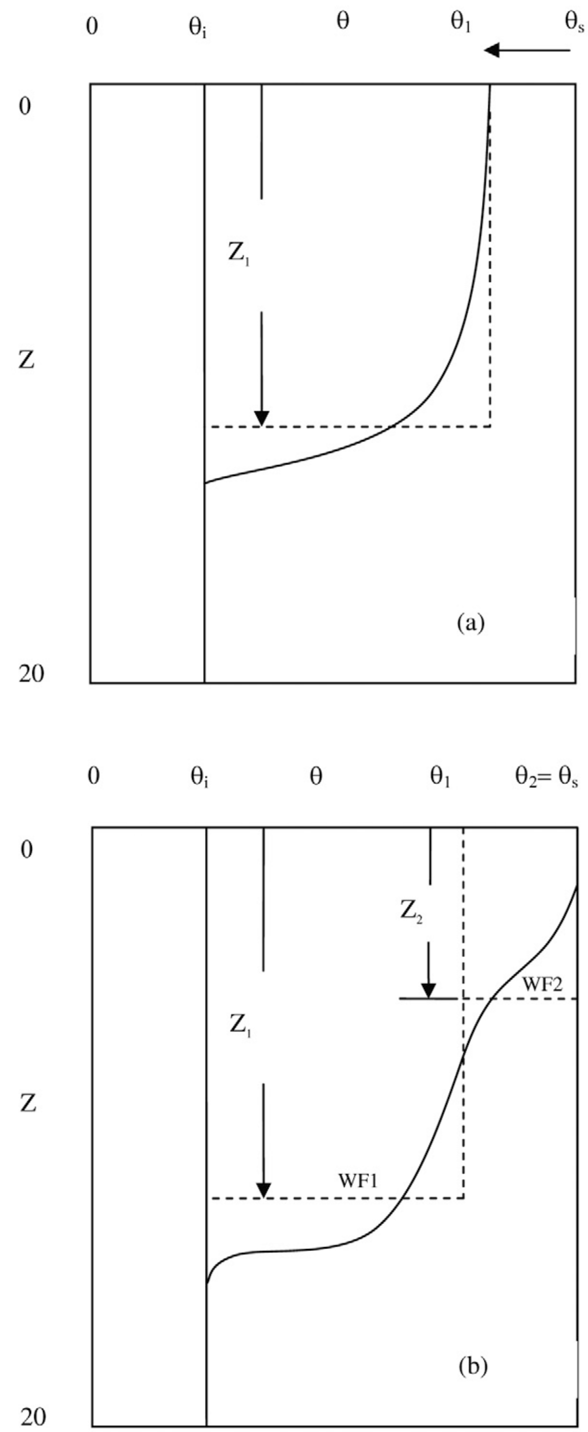

Fig. 2. The Green–Ampt with Redistribution method of calculating redistribution profiles: (a) the redistribution period of one wetting front when the rainfall rate is less than the saturated hydraulic conductivity ($r < K_s$), and (b) the infiltration and redistribution periods of two wetting fronts when $r > K_s$; see Appendix for definitions of other variables.

> ($\theta_s - \theta_1$) is often not valid during long redistribution intervals, when the error in the redistribution calculation increases.

To reduce the prediction error that occurs within the soil profile, we propose to allow the second wetting front to redistribute, and for additional fronts to form and redistribute. In the proposed modification, instead of merging the saturated front and redistributing front once the rainfall rate drops below the saturated hydraulic conductivity ($r < K_s$), the saturated front is allowed to redistribute as well. The lower front (WF1 in Fig. 2b) continues to redistribute using Eq. [1] and [3] with θ_1, θ_i, and Z_1, while the upper front (WF2) redistributes using θ_o, θ_1, and Z_2. The two fronts continue to redistribute until $Z_1 = Z_2$, at which time the two fronts are merged into one front at water content θ_o. If, instead, the rainfall rate increases so that $r > K_s$, a third front (Wetting Front 3 or WF3) is introduced. Similar to the original method, any new infiltration will supply only the uppermost wetting front (WF3) having a depth of Z_3 and will be calculated using Green–Ampt. If any of the wetting fronts' depths are equal, those fronts are merged, with the resulting front being at the higher water content. If, instead, the rainfall rate decreases so that $r < K_s$, the third front (WF3) is allowed to redistribute. The process of forming new fronts, merging fronts, and redistributing fronts is continued until the end of the simulation. Thus, the number and duration of the fronts is dependent on the soil type, the frequency of rain events, and the length of redistribution periods. The dynamic process of front formation and merging ensures that a limited number of fronts is present at a given time (we found a maximum of four for the wide range of simulations run).

Reduction of Error for Uneven Multistorm Time Series

Ogden and Saghafian (1997) reported good results for the simulations tested in their study, with a simulation time of 6 h, across all soil types defined by the USDA textural classes (Table 1). It was apparent at the end of these simulations, however, that the error in the surface water content, when compared with results from Richards' solution, increased with time. This problem was exacerbated when running longer simulation times, specifically including numerous dissimilar rain events followed by redistribution periods (i.e., $r < K_s$) and for coarser soils.

A modification is proposed here to the original GAR method to improve the accuracy with long multistorm time series. Visual comparisons of graphs of the surface water content obtained for a GAR variant coded with four redistributing wetting fronts (4GAR) against Richards' numerical solutions were used to identify a pattern in the surface water content error for long time series. The influence of several variables (rainfall intensity for each storm, length of rainfall event, number of rainfall events, length of the redistribution period, and number of redistribution periods since the beginning of the simulation) over the error trends was analyzed for all the soil textural classes (Rawls et al., 1982, 1983). A nonlinear least squares curve-fitting search procedure (TableCurve 3D, SYSTAT, Richmond, CA) was used to identify functional relationships between the error and combinations of the different factors studied for each soil type. After comparing the list of equations for each soil, a common relationship valid for all soil types was sought to provide a good fit with a minimum number of parameters.

Verification of the Modified Green–Ampt with Redistribution Model

The performance of the infiltration and redistribution methods presented here was compared with a numerical solution of Richards' equation following a procedure similar to that presented by Smith et al. (1993), Corradini et al. (1997), and Ogden and Saghafian (1997). The mechanistic-deterministic WAVE model (Vanclooster et al., 1996) was used in this comparison. This model is based on a finite differences solution of the capacitance form of Richards' equation. The WAVE model has several options for the parameterization of the unsaturated hydraulic conductivity and soil water characteristic functions, including Brooks and Corey (1964) and van Genuchten (1980).

Eleven soil types from the USDA's soil textural classification were selected (Tables 1 and 2). All the profiles were considered homogeneous, isotropic, and of semi-infinite depth (unlimited). The parameters of the MGAR (θ_s, θ_r, h_b, λ, K_s, and θ_{wp} [the wilting point water content]) were selected for each soil texture according to Rawls et al. (1982, 1983) (Table 1). Each soil was assumed to have an initial water content equal to the wilting point water content given in Table 1. The values for S_{av} and FC were calculated from those of Brooks and Corey (1964) according to Eq. [3] and the soil water retention curve at h = 333 cm, respectively. To obtain convergence of the numerical solution for all soils (in particular the coarser soils) it was necessary to use the van Genuchten (1980) model to describe the soil water retention curve and the Brooks and Corey (1964) model to describe the unsaturated hydraulic conductivity curve. The conversion of Brooks and Corey (1964) to van Genuchten (1980) parameters has been extensively investigated and shown to provide similar results for this type of

TABLE 1. Soil textures and hydraulic parameters used in conceptual model simulations.

Soil no.	USDA texture	θ_s†	θ_r†	θ_{wp}†	h_b†	λ†	K_s†	S_{av}‡	FC‡
					cm		cm h^{-1}	cm	
				From this study					
1	sand	0.417	0.020	0.033	7.26	0.694	23.56	9.62	0.048
2	loamy sand	0.401	0.035	0.055	8.69	0.553	5.98	11.96	0.084
3	sandy loam	0.412	0.041	0.095	14.66	0.378	2.18	21.53	0.155
4	loam	0.434	0.027	0.117	11.15	0.252	1.32	17.50	0.200
5	silt loam	0.486	0.015	0.133	20.79	0.234	0.68	32.96	0.261
6	sandy clay loam	0.330	0.068	0.148	28.08	0.319	0.30	42.43	0.187
7	clay loam	0.390	0.075	0.197	25.89	0.242	0.20	40.89	0.245
8	silty clay loam	0.432	0.040	0.208	32.56	0.177	0.20	53.83	0.300
9	sandy clay	0.321	0.109	0.239	29.17	0.223	0.12	46.65	0.232
10	silty clay	0.423	0.056	0.250	34.19	0.150	0.10	57.77	0.317
11	clay	0.385	0.090	0.272	37.30	0.165	0.06	62.25	0.296
				From Corradini et al. (1997)					
B	clay loam	0.333	0.123	–	80.00	0.200	0.04	65.00	0.170
SL	sandy loam	0.412	0.041	–	30.00	0.500	2.50	21.00	0.152

† Selected according to Rawls et al. (1982, 1983): θ_s, saturated volumetric water content; θ_r, residual volumetric water content; θ_{wp}, volumetric water content at the wilting point; h_b, bubbling pressure; λ, pore size distribution index; K_s, saturated hydraulic conductivity.
‡ Calculated from Brooks and Corey (1964) equations: S_{av}, suction at the wetting front; FC, field capacity.

TABLE 2. The van Genuchten water characteristic curve parameters of saturated and residual volumetric water content θ_s and θ_r and shape parameters α, *n*, and *m* used in Richards' numerical solutions.

Soil no.	USDA texture	θ_s	θ_r	α†	*n*†	*m*†
1	sand	0.417	0.020	0.1310	82.5535	0.0085
2	loamy sand	0.401	0.035	0.1075	68.6080	0.0082
3	sandy loam	0.412	0.041	0.0608	66.4880	0.0059
4	loam	0.434	0.027	0.0804	48.2744	0.0054
5	silt loam	0.486	0.015	0.0448	72.0750	0.0033
6	sandy clay loam	0.330	0.068	0.0332	60.8529	0.0053
7	clay loam	0.390	0.075	0.0363	84.9396	0.0029
8	silty clay loam	0.432	0.040	0.0295	111.7583	0.0016
9	sandy clay	0.321	0.109	0.0326	93.2147	0.0024
10	silty clay	0.423	0.056	0.0283	128.0623	0.0012
11	clay	0.385	0.090	0.0259	126.4091	0.0013

† Derived using the RETC program, in conjunction with the Brooks and Corey hydraulic conductivity parameters in Table 1.

TABLE 3. Rain intensities, *A* and *B*, for each soil type as used in the simulated multistorm time series.

Soil no.	USDA texture	Rainfall intensity	
		A	*B*
		cm h^{-1}	
1	sand	30.0	31.0
2	loamy sand	15.0	16.0
3	sandy loam	7.0	8.0
4	loam	4.0	5.0
5	silt loam	4.0	5.0
6	sandy clay loam	2.0	3.0
7	clay loam	2.0	3.0
8	silty clay loam	2.01	3.01
9	sandy clay	1.0	2.0
10	silty clay	2.0	3.0
11	clay	1.0	2.0

problem (Stankovich and Lockington, 1995; Lenhard et al., 1989; van Genuchten and Nielsen, 1985; Morel-Seytoux et al., 1996). The corresponding van Genuchten parameters were obtained using the RETC program (van Genuchten et al., 1991) based on the curves described by the Brooks and Corey parameters (Table 2). The soil hydraulic functions obtained for both models were compared and were found to be close matches of each other, thus ensuring the validity of the approach.

For the finite-difference solution in WAVE, the soil profile was discretized in 5-mm segments and the initial condition for soil water content was set to θ_{wp} (Table 1) for the entire profile, $\theta(x,0) = \theta_i$ for $x > 0$. For each soil type, the simulation consisted of eight uneven precipitation events, each followed by a redistribution period (where $r = 0.0$ cm h^{-1}) for a total simulation time of 365 h. The duration and intensity of each rain event and duration of the redistribution periods varied during the simulation. To compare the performance of the modified model against the original GAR results obtained by Ogden and Saghafian (1997), we selected the same rainfall intensities to guarantee surface ponding in each profile, with the exception of sand for which the rate was decreased to reduce the high water balance error that occurred during the numerical simulation. It was assumed that the surface rainfall excess produced was lost instantly to overland flow (surface storage, $s = 0$). The rain events used for both the Richards' and approximate solutions are described in Tables 3 and 4.

One of the most frequent applications of hydrologic models of the unsaturated zone is the prediction of the soil water content for a desired observation layer within the soil profile, corresponding, for example, to the plant root zone, to biochemically active horizons, etc. In our tests, we computed the average soil water content for the observation layers by considering the rectangular soil water profiles proposed by the method and calculating a weighted average of the different wetting fronts (with corresponding depths and water contents) contained within each of the prescribed observation layers.

A final test of the accuracy of the MGAR model was made by comparing the independent results obtained by Corradini et al. (1997) in their Events 1 to 3 (see Table 5 and Fig. 9–11 in Corradini et al., 1997). The parameters for their two soils can be found in Table 1.

The goodness-of-fit of the entire simulation against the results of Richards' equation was evaluated using the Nash and Sutcliffe (1970) coefficient of efficiency and the RMSE. Four simulation outputs were selected: surface water content, θ_o, cumulative infiltration, and average water content for two layers of interest, 0 to 50 and 50 to 100 cm.

Results and Discussion

A computer code was developed following the original GAR formulation as well as a variant that included the proposed multiple redistributing wetting fronts scheme. Additional details and sample code can be found in Gowdish (2007). The surface water content estimation error of the 4GAR formulation, $\varepsilon_{\theta o}$, was calculated as the absolute value of the difference between 4GAR and Richards' θ_o outputs for each time step (Fig. 3). A general trend in the error produced by 4GAR with respect to Richards' solution was calculated using fast Fourier transform smoothing (Press et al., 1989) across all soil types studied (Fig. 3). Although the general trend in error increased with every redistribution period, the error tended to decrease at a decreasing rate with time within each redistribution period. In addition, while the same pattern

TABLE 4. Multistorm time series used for model simulation and rainfall intensities *A* and *B* for each soil type from Table 3.

Rain event	t_{start}†	Rainfall rate				
		t_{start}	t_{start} + 1 h	t_{start} + 2 h	t_{start} + 3 h	t_{start} + 4 h
	h	cm h^{-1}				
1	0	*A*	*A*	0.0	0.0	0.0
2	71	*A*	*B*	*A*	*A*	0.0
3	92	*A*	*A*	0.0	0.0	0.0
4	139	*B*	*A*	0.0	0.0	0.0
5	163	*A*	0.0	0.0	0.0	0.0
6	223	*A*	*A*	*B*	*B*	*A*
7	258	*B*	*B*	0.0	0.0	0.0
8	342	*A*	*B*	*B*	0.0	0.0

† t_{start} , the time since the beginning of the simulation when a new precipitation period starts after a hiatus.

TABLE 5. Parameters for the Γ redistribution coefficient (Eq. [5]).

i	b_i	c_i	d_i	Adjusted R^2
1	4.2952	154.6101	−1.0	0.998
2	0.0020	-0.0010	0.5	0.988
3	−14.0032	-61.5429	−1.0	0.969

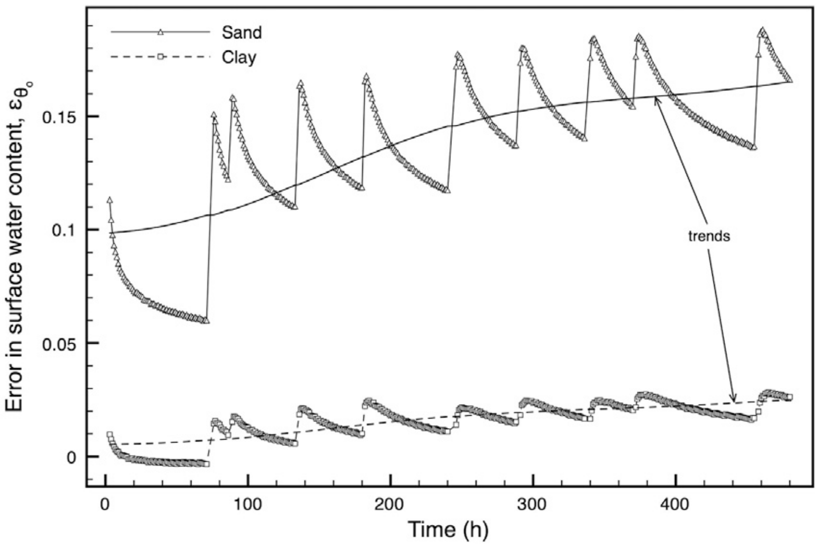

FIG. 3. Two examples, sand and clay, showing the actual error and the general error trend observed in surface water content between a Green–Ampt with Redistribution variant coded with four redistributing wetting fronts (4GAR) and Richards' solution.

was observed for all of the soil types tested, the magnitude of the error depended on the specific soil. An extensive analysis of the error showed that three main variables, redistribution time (T_R), redistribution number (N_R), and saturated hydraulic conductivity (K_s), largely controlled the observed error for all soil types. The redistribution number corresponds to the redistribution period; i.e., if there were three redistribution periods within the simulation, then N_R = 1, 2, and 3 for the first, second, and third redistribution periods, respectively. The redistribution time does not correspond to the general simulation time, but rather the local time within each redistribution period. For example, if a rain event ceases, marking the start of a redistribution period, at simulation time t = 10.0 h, the corresponding redistribution time T_R = 0.0 h. Within that redistribution event, if t = 11 h then T_R = 1.0 h, etc. When there are multiple wetting fronts redistributing, each wetting front i has its own N_{Ri} and T_{Ri}. Each wetting front's N_{Ri} would correspond to the simulation redistribution period in which it began redistributing and its T_{Ri} would correspond to the time within that N_{Ri}. For instance, if the first redistribution period began at simulation time t = 10.0 h, the corresponding N_{R1} would be 1 and the T_{R1} would be 0.0 h for WF1. If the next redistribution period began at simulation time t = 20.0 h and there is a second wetting front, WF2, then the corresponding N_{R2} and T_{R2} would be 2 and 0.0 h, respectively, while the corresponding N_{R1} and T_{R1} for WF1 would be 1 and 10.0 h, since it began redistributing in the first period and has been redistributing for 10.0 h. Similarly, if a third wetting front, WF3, begins redistributing, the same procedure is used. If any of the fronts merge while they are redistributing, the corresponding N_{Ri} and T_{Ri} of the merged front are those of upper wetting front.

A common equation of the three variables to predict the error was identified that provided a good response across all soil types (adjusted R^2 > 0.86). Since the error equation depends on redistribution characteristics, it was named here the *redistribution coefficient*, Γ:

$$\Gamma = a_1 + a_2 \ln\left(T_R\right) + a_3/N_R \qquad [5]$$

$$\text{with } a_i = \left(b_i + c_i K_s^{d_i}\right)^{(-1)^i} \quad \text{for } i = 1, 2, 3 \qquad [6]$$

where a_1, a_2, and a_3 strongly depend (adjusted R^2 > 0.969) on the saturated hydraulic conductivity (mm h^{-1}) of the soil for all soils tested (Fig. 4), and the values of parameters b_i, c_i, and d_i are given in Table 5.

During the simulation, the proposed redistribution coefficient was calculated at every time step using the appropriate N_R and T_R. For the first time step in which a wetting front redistributes, the change in water content was calculated using Eq. [1] and the current water content, i.e., the saturated water content for this first time step. This procedure was repeated thereafter for every time step within that redistribution period using the uncorrected water content calculation.

Figure 5 is a graphical comparison of the evolution of the relative surface water content during the simulations for the same six USDA soil textural types—sand, sandy loam, silt loam, clay loam, sandy clay, and clay—used by Ogden and Saghafian (1997). The GAR plot is a graph of the original methodology presented by Ogden and Saghafian (1997), whereas the MGAR plot is a graph of the modified methodology utilizing the multiple wetting

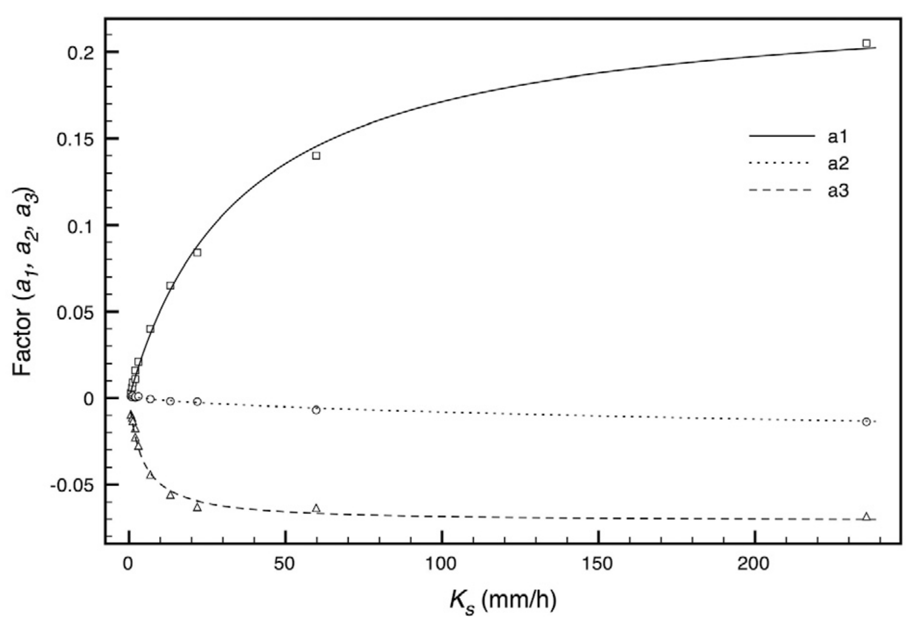

FIG. 4. Relationship between saturated hydraulic conductivity, K_s, and the parameters a_1, a_2, and a_3 used in the calculation of the redistribution coefficient, Γ (Eq. [5]). Symbols indicate values obtained for each soil type and lines are the values obtained with the auxiliary Eq. [6] (adjusted R^2 > 0.97).

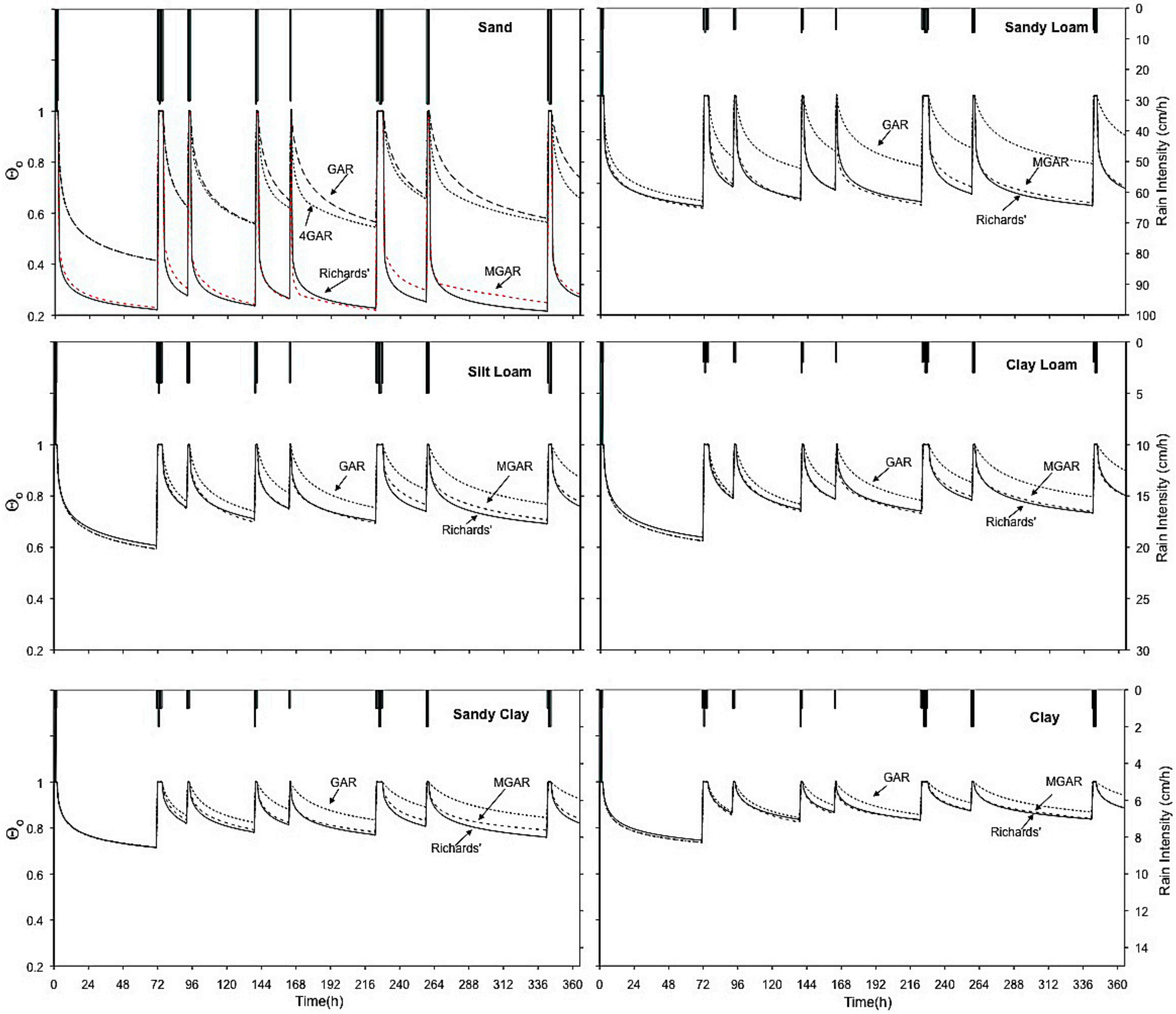

Fig. 5. Comparison of soil surface relative water content (Θ_o) vs. time for the Richards', Green–Ampt with Redistribution (GAR), and modified GAR (MGAR) solutions for six selected soils. Vertical bars on top represent rainfall intensities. Results from a GAR variant coded with four redistributing wetting fronts but no redistribution coefficient (4GAR) are also included for the sandy soil to visually assess the effect of the addition of multiple redistributing wetting fronts on the solution.

front scheme and the redistribution coefficient. It can be observed by visual inspection that in all cases, GAR diverged significantly from the numerical solution, while MGAR closely matched it. A comparison between these results and those obtained with 4GAR, i.e., when only multiple redistributing fronts are allowed (no redistribution coefficient), is presented for sand (Fig. 5). This figure illustrates the relative importance of the two modifications implemented in MGAR. In general, the improvements obtained with just the multiple-front scheme (4GAR) are small compared with those obtained by the addition of the redistribution coefficient. Note that the 4GAR vs. GAR differences are smaller for all other soils (results not shown).

Statistics for all four outputs of interest are presented in Table 6. For all soil textural classes, the coefficients of efficiency for surface water content errors ranged from −3.659 to 0.846 for GAR and from 0.935 to 0.991 for MGAR, showing the vast improvement obtained using the proposed redistribution coefficient across all soil types, especially the sandy soils. The cumulative infiltration errors remained similarly low for both methods.

The results of the average water content for the two layers, θ_{0-50} and θ_{50-100}, show an improvement in the prediction for the top layer for all soil types. In addition to the significant improvement to the surface water content and average water content of the top layer, the predictions of the lower layer were maintained in relation to the same results of the original method. The largest errors in both layers occurred in the coarse soil textures due to rapid drying at the soil surface related to high hydraulic conductivities and the Green–Ampt assumption of a flat ("piston") advancing front.

An additional benefit of using the multiple redistributing wetting front scheme over the original scheme can also be seen by comparing the shapes of the water content profiles for a generic test case after four redistribution events in a clay soil (Fig. 6). The improvement obtained with the multiple redistributing fronts

Table 6. Goodness of fit of simulations comparing the volumetric water content values at the surface (θ_o) and at 0- to 50- and 50- to 100-cm depth (θ_{0-50} and θ_{50-100}, respectively) determined by the Green–Ampt with Redistribution (GAR) and the modified GAR (MGAR) models against Richards' solution. Values given are coefficients of efficiency with RMSE values in parentheses.

Soil no.	Soil texture	θ_o		θ_{0-50}		θ_{50-100}		Cumulative infiltration	
		GAR	MGAR	GAR	MGAR	GAR	MGAR	GAR	MGAR
1	sand	−3.659 (0.150)	0.975 (0.011)	−2.463 (0.125)	0.888 (0.023)	−1.344 (0.098)	0.385 (0.050)	1.000 (2.184)	0.998 (6.474)
2	loamy sand	−2.675 (0.105)	0.977 (0.008)	−1.265 (0.078)	0.560 (0.034)	−0.301 (0.053)	−1.020 (0.06)	0.996 (2.912)	0.994 (3.831)
3	sandy loam	−1.273 (0.064)	0.980 (0.006)	−0.206 (0.044)	0.637 (0.024)	0.886 (0.025)	0.626 (0.045)	0.992 (2.049)	0.996 (1.559)
4	loam	−0.598 (0.047)	0.978 (0.006)	0.617 (0.028)	0.822 (0.019)	0.975 (0.015)	0.872 (0.034)	0.998 (0.677)	0.995 (1.041)
5	silt loam	0.364 (0.032)	0.971 (0.007)	0.931 (0.017)	0.954 (0.013)	0.982 (0.015)	0.962 (0.022)	1.000 (0.219)	0.925 (1.005)
6	sandy clay loam	0.219 (0.022)	0.935 (0.006)	0.862 (0.011)	0.955 (0.006)	0.961 (0.011)	0.957 (0.011)	0.998 (0.244)	0.997 (0.331)
7	clay loam	0.533 (0.018)	0.961 (0.005)	0.937 (0.009)	0.978 (0.005)	0.968 (0.010)	0.978 (0.008)	1.000 (0.103)	0.994 (0.375)
8	silty clay loam	0.795 (0.013)	0.988 (0.006)	0.961 (0.009)	0.985 (0.006)	0.972 (0.011)	0.979 (0.009)	0.998 (0.265)	0.987 (0.608)
9	sandy clay	−0.002 (0.014)	0.939 (0.004)	0.628 (0.008)	0.923 (0.004)	0.870 (0.008)	0.883 (0.007)	0.996 (0.177)	0.995 (0.209)
10	silty clay	0.846 (0.009)	0.991 (0.003)	0.939 (0.009)	0.993 (0.003)	0.969 (0.007)	0.968 (0.007)	0.997 (0.192)	0.986 (0.401)
11	clay	0.759 (0.009)	0.989 (0.002)	0.883 (0.008)	0.984 (0.003)	0.941 (0.006)	0.939 (0.006)	0.998 (0.106)	0.980 (0.322)

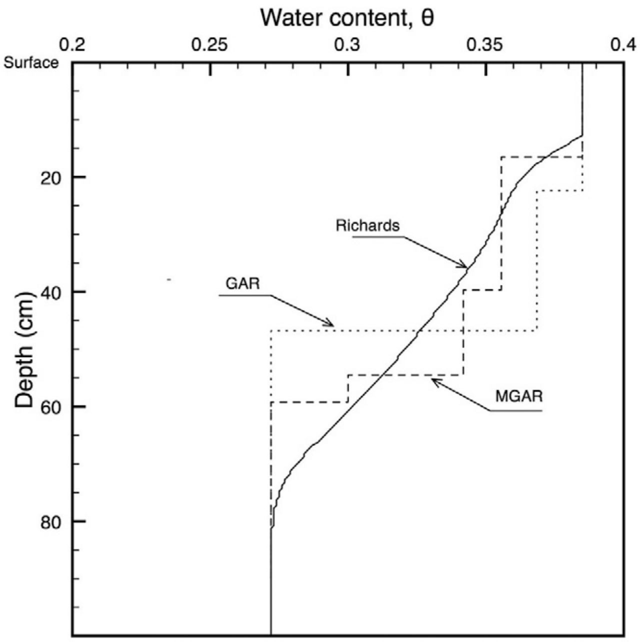

Fig. 6. Example of soil water content profiles obtained by the Richards', Green–Ampt with Redistribution (GAR), and modified GAR (MGAR) solutions for a clay soil after a generic run with four rainfall events.

(four were formed in the test case) is related to the integration error caused by approximating the area under a curve using rectangles of finite width. From the concept of Riemann's sum of integrals, we know that using narrower rectangles, and therefore more rectangles, yields a better approximation. Therefore, the MGAR scheme, which allows a greater number of rectangular fronts than the GAR scheme, better approximates the actual wetted profile.

Figure 7 presents the comparison of results obtained by MGAR against those obtained by Corradini et al. (1997). While all methods performed well against the Richards' solution for the clay soil (Events 1 and 2), it should also be observed how MGAR controlled the increasing error with time shown by the conceptual model revised by Corradini et al. (1997). For the case of the sandy loam soil (Event 3), the error decrease is dramatic (errors in surface water content at 7 and 13 h of <1 and 5%, respectively, compared with 11 and 16% obtained in the original study). Since these events were selected by the researchers to check some of the main features of the revised method (i.e., the transition from single to compound wetting fronts and again to single, and wetting front consolidation), this confirms the excellent behavior of the MGAR model for soils where previous formulations of the conceptual model produced inaccurate results.

In terms of computational efficiency, the proposed method was found to be consistently faster that the Richards solutions, in line with the result reported by Corradini et al. (1997). This was expected since the calculation of the redistribution coefficient has negligible overhead in the calculations with respect to the initial version.

Summary and Conclusions

When formulating a simplified but physically based soil water simulation model, it is desirable that both the infiltration and redistribution components of the model be based on the same underlying assumptions. This not only ensures physical and numerical consistency of the model, but can reduce the number of inputs needed by the model and avoid overparameterization issues often found in mixed models. The GAR method meets this requirement and is able to accurately simulate the infiltration and soil water redistribution cycles caused by multistorm time series. The method requires few parameters and is computationally efficient and robust; however, the original results show that for soils with larger saturated hydraulic conductivity values, the error in the prediction of the surface water content is significant, and also that when the method is applied to a long period of uneven storms, there is an increasing divergence from the actual surface water content after subsequent redistributions.

Modifications to the original GAR method were developed and tested for improving the estimation of soil water infiltration and redistribution for a wide range of soils. The first modification expanded the number of redistributing wetting fronts from one or two to any number, providing a better physical representation to predict the average water content for a desired observation soil layer. Since wetting fronts are formed and merge dynamically, it was found that for our wide range of simulations, only four fronts were present at the same time. The second modification consisted of the application of a correction factor, the redistribution coefficient Γ, derived from an error analysis of the method as a function of the redistribution number (N_R), redistribution time (T_R), and saturated hydraulic conductivity (K_s). A comparison of a numerical solution of Richards' equation against the GAR and MGAR methods showed that the latter is markedly better

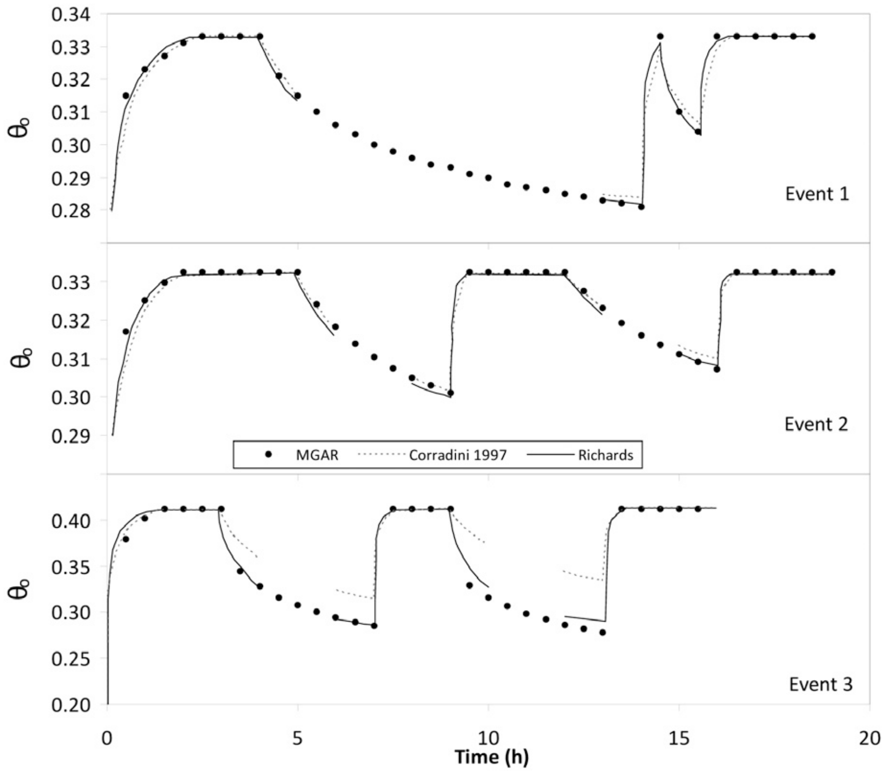

FIG. 7. Comparison of the Richards' and the modified Green–Ampt with Redistribution (MGAR) solutions with the results of Corradini et al. (1997) for their rainfall events 1 to 3.

at predicting the surface water content while maintaining good cumulative infiltration predictions. In addition, the ability of MGAR to predict the average water content for a desired observation soil layer was also found to provide good results against Richards' numerical solution values.

While the MGAR approach is limited in that it assumes a homogeneous soil and a rectangular wetting profile and does not consider hysteresis or water table effects, it provides good agreement with Richards' equation. Among the advantages of the method, besides its simplicity and computational efficiency set against Richards' solution, are its physical basis (set against other more empirical methods), its robustness, and its ability to provide good results for infiltration and redistribution for uneven multistorm precipitation time series.

Appendix

- F cumulative infiltration
- FC field capacity [$L^3\ L^{-3}$]
- h_b bubbling pressure [L]
- K_s saturated hydraulic conductivity [$L\ T^{-1}$]
- n van Genuchten shape parameter
- r rainfall rate [$L\ T^{-1}$]
- S_{av} suction at the wetting front [L]
- t simulation time [T]
- WF1, WF2 wetting fronts 1 and 2 of the GAR method
- WF3 wetting front 3 of the MGAR method
- Z wetting front depth [L]
- Z_m weighted average of Wetting Fronts 1 and 2 with respect to their soil water contents
- Z_1, Z_2, Z_3 depths of Wetting Fronts 1, 2, and 3, respectively [L]
- α van Genuchten shape parameter [L^{-1}]
- λ pore size distribution index
- θ_1, θ_2 water contents of Wetting Fronts 1 and 2, respectively [$L^3\ L^{-3}$]
- θ_{0-50} average water contents at 0- to 50-cm depth [$L^3\ L^{-3}$]
- θ_{50-100} average water contents at 50 to 100-cm depth [$L^3\ L^{-3}$]
- θ_i initial water content [$L^3\ L^{-3}$]
- θ_o surface water content [$L^3\ L^{-3}$]
- Θ_o relative volumetric water content of the soil at the surface
- Θ_i initial relative volumetric water content of the soil profile
- θ_r residual water content [$L^3\ L^{-3}$]
- θ_s saturated water content [$L^3\ L^{-3}$]
- θ_{wp} wilting point water content [$L^3\ L^{-3}$]

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