One main goal of stress analysis is to determine the point within a load-carrying member that is subjected to the highest stress level. The orientation of the stress element is critical, and it must be aligned with specified axes on the member, usually called x, y, and z.

Tensile and compressive stresses, called normal stresses, act perpendicular to the opposite faces of the stress element. Tensile stresses pull on the element. Compressive stresses crush the element.
Stress Elements for 3 Types of Stresses

Shear stresses are created by direct shear, vertical shear in beams, or torsion.

There is a tendency to cut the element by exerting a stress downward on one face while simultaneously exerting a stress upward on the opposite, parallel face.

Sign Convention for Shear Stresses

- Positive shear stresses rotate the element in a clockwise direction.
- Negative shear stresses rotate the element in a counterclockwise direction.
- A double subscript notation is used to denote shear stresses in a plane. For example, drawn for the y-z plane, the pair of shear stresses, $\tau_{yz}$, indicates a shear stress acting on the element face that is perpendicular to the y-axis and parallel to the z-axis.
- So, $\tau_{yz}$ acts on the face that is perpendicular to the y-axis and parallel to the x-axis. In this example, $\tau_{yz}$ is positive and $\tau_{xz}$ is negative.
Direct Stresses: Tension and Compression

- Stress = the internal resistance offered by a unit area of a material to an externally applied load. Normal stresses (σ) can be tensile (positive) or compressive (negative).
- The magnitude of the stress can be calculated from the direct stress formula:
  - σ = force / area = F/A
  - (US Units) lb / in² = psi ; (SI Units) N / m² = pascal = Pa

Example Problem 3-1: A webbed flange of 9,000 lb is applied to a 12-mm-diameter round bar, as shown in Figure 3-1. Compute the direct tensile stress in the bar.

- Objective: Compute the tensile stress in the round bar.
- Given: Load = F = 9000 lb, diameter = D = 12 mm.
- Analysis: Use the direct tensile stress formula, Equation (3-1) σ = F/A. Compute the cross-sectional area from A = πD²/4.
- Result: \[ σ = \frac{F}{A} = \frac{9000}{\pi \times (12/2)^2} = 123 \text{ MPa} \]
- Comment: The results are shown in stress diagrams as in Figure 3-6, which can be taken to be equivalent within the bar because, ideally, the stress is uniform in any cross section. The curve form of the data is shown in Figure 3-5(a).

Direct Stresses: Tension and Compression con’t

- The conditions on the use of the previous equation are as follows:
  1. The load-carrying member must be straight
  2. The line of action of the load must pass through the centroid of the cross-section of the member
  3. The member must be of uniform cross section near where the stress is being computed
  4. The material must be homogeneous and isotropic
  5. In the case of compression members, the member must be short to prevent buckling.
Deformation Under Direct Axial Loading

The following formula computes the stretch due to a direct axial tensile load or the shortening due to a direct axial compressive load:

\[ \delta = \frac{FL}{EA} \]  

(3-2)

where \( \delta \) = total deformation of the member carrying the axial load  
\( F \) = direct axial load  
\( L \) = original total length of the member  
\( E \) = modulus of elasticity of the material  
\( A \) = cross-sectional area of the member  

Noting that \( \sigma = \frac{F}{A} \), we can also compute the deformation from:

\[ \delta = \frac{nFL}{EA} \]  

(3-3)

Direct Shear Stress

- Direct shear stress occurs when the applied force cuts through the member like scissors, or when a punch and a die are used to punch a slug of material from a sheet.
- The method of computing direct shear stress is similar to the method used for computing direct tensile stress. Both methods assume the applied force is uniformly distributed across the cross section of the part that is resisting the force.
- \( \tau = \frac{\text{shearing force}}{\text{area in shear}} = \frac{F}{A_s} \)
### Relationship Among Torque, Power, and Rotational Speed

- The relationship among the power (P), the rotational speed (n), and the torque (T) in a shaft is described by the equation:
  - $T = \frac{P}{n}$ (SI) or $T = 63,000 \frac{P}{n}$ (English)
- In SI units, power is expressed in:
  - watt (W) or its equivalent
  - newton meter per second (N*m/s)
- Rotational speed is in radians per second (rad/s)

### Torsional Shear Stress

- A torque causes a rotation of one part of the member relative to another, which leads to shear stress.
- In torsional shear, the distribution of stress is not uniform across the cross section.
- Most common case: a round circular shaft transmitting power.

### Torsional Shear Stress Formula

- When the outer surface of a solid round shaft is subjected to torque, it experiences the greatest shearing strain and the largest torsional shear stress.
  - $\tau_{\text{max}} = \frac{T\cdot c}{J}$
  - Where $c =$ radius of the shaft to its outside surface
  - $J =$ polar moment of inertia
Stress Distribution in a Solid Shaft

General Formula

- To calculate the torsional shear stress at some point inside the shaft, the formula used is:
  - \( \tau = \frac{T \cdot r}{J} \)
  - Where \( r \) = radial distance from the center of the shaft to the point of interest.
- In hollow shafts, the maximum shear stress occurs at the outer surface.
- The hollow shaft is more efficient because the entire cross section carries a relatively high stress level.

Example Problem 3-8

**Objective:** Compute the maximum torsional shear stress in a shaft having a diameter of 10 mm when it carries a torque of 4.10 N·m.

**Given:**
- Torque = \( T = 4.10 \text{N} \cdot \text{m} \)
- Shaft diameter = \( D = 10 \text{mm} \)
- \( r = \frac{D}{2} = 5.0 \text{mm} \)

**Analysis:** Use Equation (3-7) to compute the torsional shear stress: \( \tau_{\text{max}} = \frac{T \cdot r}{J} \).

- \( J \) is the polar moment of inertia for the shaft: \( J = \pi D^4/32 \) (see Appendix 1).

**Results:**
- \( J = \pi (5.0 \text{mm})^4/32 = 982 \text{mm}^4 \)
- \( \tau_{\text{max}} = \frac{4.10 \text{N} \cdot \text{m} \cdot 5.0 \text{mm}}{982 \text{mm}^4} = 20.9 \text{N/mm}^2 = 20.9 \text{MPa} \)

**Comment:** The maximum torsional shear stress occurs at the outer surface of the shaft around its center circumference.
Stress Distribution in a Hollow Shaft

Polar Section Modulus

A design simplification uses the polar section modulus,

\[ Z_p = J / c \]

The equation for the maximum torsional shear stress is:

\[ \tau_{\text{max}} = T / Z_p \]

When one cross-section is rotated relative to other cross-sections due to torque, the angle of twist is computed from:

\[ \theta = TL / GJ \]

Where \( \theta \) = angle of twist (radians)
\( L \) = length of the shaft over which the angle is being computed
\( G \) = modulus of elasticity of the shaft material in shear

Vertical Shearing Stress

A beam carrying loads transverse to its axis will experience shearing forces (V).

To analyze the beams, you compute the variation in shearing force across the entire length of the beam and draw the shearing force diagram.

The resulting vertical shearing stress \( \tau \) is computed by:

\[ \tau = VQ / It \]

Where \( I \) = rectangular moment of inertia of the cross section of the beam
\( t \) = thickness of the section at the place where the shearing stress is to be computed
\( Q \) = first moment, with respect to the overall centroidal axis
Vertical Shearing Stress con’t

To calculate the value of Q:
- \( Q = A_p y \)
- Where \( A_p \) = that part of the area of the section above the place where the stress is to be computed
- \( y \) = distance from the neutral axis of the section to the centroid of the area \( A_p \).

Vertical Shearing Stress con’t

For most section shapes, the maximum vertical shearing stress occurs at the centroidal axis.

\[ \text{FIGURE 5-14} \quad \text{Diagrams of} \ A_p \text{ and } y \text{ used to compute } Q \text{ for these shapes} \]

\[ Mott, \text{Machine Elements in Mechanical Design, 2003} \]
Special Shearing Stress Formulas

- The most common cross sections have unique formulas for finding the maximum vertical shearing stress:
  \[ \tau_{\text{max}} = \frac{3V}{2A} \quad \text{(for rectangle)} \]
  \[ \tau_{\text{max}} = \frac{4V}{3A} \quad \text{(for circle)} \]

**Stress Due to Bending**

- A beam is a member that carries loads transverse to its axis, producing bending moments in the beam. Bending stresses are normal stresses (tensile or compressive).
- The maximum bending stress in a beam cross section occurs in the part farthest from the neutral axis of the section.
  \[ \sigma = \frac{M*c}{I} \]

Where \( M \) = magnitude of bending moment at section
\[ I = \text{moment of inertia of section w.r.t neutral axis} \]
\[ c = \text{distance from n.a. to outer most fiber} \]
Stress Due to Bending con’t

- The value varies from zero at the neutral axis, to the maximum tensile stress on one side of the neutral axis, to the maximum compressive stress on the other side.
- Positive bending only occurs when the deflected shape of the beam is concave upward, resulting in compression on the upper part of the cross section and tension on the lower part.
- Negative bending causes the beam to be concave downward.

Stress Due to Bending con’t

- The flexure formula was developed subject to the following conditions:
  1. The beam must be in pure bending. Shearing stresses must be zero or negligible. No axial loads present.
  2. The beam must not twist or be subjected to a torsional load.
  3. The material of the beam must obey Hooke’s law.
  4. The modulus of elasticity of the material must be the same in both tension and compression.

Stress Due to Bending con’t

5. The beam is initially straight and has a constant cross section
6. Any plane cross section of the beam remains plane during bending.
7. No part of the beam shape fails because of local buckling or wrinkling.
For design, it is convenient to define the term section modulus, \( S \), as

\[
S = \frac{I}{c} \quad (3-23)
\]

The flexure formula then becomes

\[
\sigma = \frac{M}{S} \quad (3-24)
\]

Since \( I \) and \( c \) are geometrical properties of the cross section of the beam, \( S \) is also. Then, in design, it is usual to define a design stress, \( \sigma_d \), and, with the bending moment known, solve for \( S \):

\[
S = \frac{M}{\sigma_d} \quad (3-25)
\]

For the beam shown in Figure 3-16, the load \( F \) due to the pipe is 12,000 lb. The distances are \( a = 4 \) ft and \( b = 6 \) ft. Determine the required section modulus for the beam to limit the stress due to bending to 30,000 psi; the environmental design service for a typical structural steel in static bending.

**Objective**

Compute the required section modulus \( S \) for the beam in Figure 3-16.

**Given**

Length: Overall length \( L = 30 \) ft; \( a = 4 \) ft; \( b = 6 \) ft.

Load: \( F = 12,000 \) lb.

Design stress: \( \sigma_d = 30,000 \) psi.

**Analysis**

Use Equation (3-25) to compute the required section modulus \( S \). Compute the maximum bending moment that occurs at the point of application of the load using the formula shown in Part (d) of Figure 3-16.

**Results**

\[
M_{\text{max}} = \frac{Fr}{d} = \frac{12,000 \times 6(4)}{0} = 28,800 \text{ ft-lb}
\]

\[
S = \frac{M}{\sigma_d} = \frac{28,800 \text{ ft-lb}}{30,000 \text{ psi}} = 0.96 \text{ in}^3
\]

**Conclusion**

A beam section can now be selected from Tables A16-3 and A16-4 that has at least this value for the section modulus. The lightest section, typically preferred, is the W21X23 wide-flange shape with \( S = 11.8 \text{ in}^3 \).
Combined Normal Stresses

When the same cross section of a load-carrying member experiences both a direct normal stress and a bending stress, the resulting normal stress can be computed by the method of superposition.

\[ \sigma = \pm \left( \frac{Mc}{I} \right) \pm \left( \frac{F}{A} \right) \]

Where tensile stresses are positive and compressive stresses are negative.

Combined Normal Stresses con’t

An example of this situation involves a beam subjected to a load applied downward and to the right through a bracket below the beam. The load’s horizontal and vertical components show that the effect can be broken into three parts:

1. The vertical component places the beam in bending with tension on the top and compression in the bottom.
2. The horizontal component, because it acts away from the neutral axis of the beam, causes bending with tension on the bottom and compression on the top.
3. The horizontal component causes direct tensile stress across the entire cross section.

Combined Normal Stresses con’t

1. Prepare the shearing force and bending moment diagrams.
2. Combine the effects of the bending stress and the direct tensile stress at any point.