CWR 6536
Stochastic Subsurface Hydrology

Dr. Wendy Graham
570 Weil Hall
Phone 392-5893 x2113
E-mail: wgraham@ufl.edu
Web page: www.abe.ufl.edu/graham
Goal of Stochastic Hydrology

- Develop analytical tools to systematically deal with uncertainty and spatial variability in hydrologic systems
- Examples of variable driving parameters and processes include rainfall rates, soil properties, aquifer properties
- Examples of resulting variable hydrologic processes include water contents, piezometric heads, water flow rates, solute concentrations
Uncertainty vs Variability

- Uncertainty - incomplete knowledge of the hydrologic system can result in model error and parameter error
- Spatial Variability - objective value (but often unknown). Magnitude of spatial variability depends on geology for example
- Temporal Variability - future values of rainfall, evapotranspiration etc unknown
- All contribute to accuracy problems when predicting the behavior of hydrologic systems
Natural Variability

• It is widely recognized that natural earth materials are quite heterogeneous in their hydrologic properties
• However variation is not completely disordered in space. Higher than average and lower than average values tend to occur in zones
• Examples…. 
Fig. 1. Photograph showing natural heterogeneity of a sand and gravel deposit in Switzerland (photo by E. Trüeb).
Fig. 2. Permeability (millidarcy) and porosity data from laboratory analyses of cores from a borehole in the Mt. Simon aquifer in Illinois [Bakr, 1976] (1 foot = 30.48 cm).
Figure 1.4. Infiltration rate of soil surface observed at 25-ft intervals in recent alluvain of the Rio Grande near Socorro, New Mexico (data from Gelhar et al., 1983).
Fig. 5. Comparison of hydraulic conductivity profiles for cores separated by a 1-m horizontal distance.

Sudicky at al, WATER RESOURCES RESEARCH, VOL. 22, No. 13, Pages 2069-2082, December 1986
Fig. 6. Location of measurements and distribution of $-\ln (K)$ along $A - A'$ (contour interval = 0.5; vertical exaggeration = 2; $K < 10^{-3}$ cm/s in stippled zones).
### TABLE 2. Variances and Correlation Scales for Log Hydraulic Conductivity or Log Transmissivity

<table>
<thead>
<tr>
<th>Source</th>
<th>Medium</th>
<th>$\sigma_f$</th>
<th>Correlation Scale, m</th>
<th>Overall Scale, m</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bakr [1976]</td>
<td>sandstone aquifer</td>
<td>1.5–2.2</td>
<td>0.3–1.0 V</td>
<td>100</td>
</tr>
<tr>
<td>Smith [1978]</td>
<td>outwash sand</td>
<td>0.8</td>
<td>0.4 V</td>
<td>30</td>
</tr>
<tr>
<td>Delhomme [1979]</td>
<td>limestone aquifer</td>
<td>2.3</td>
<td>6300 H</td>
<td>30,000</td>
</tr>
<tr>
<td>Binsariti [1980]</td>
<td>basin fill aquifer</td>
<td>1.0</td>
<td>800 H</td>
<td>20,000</td>
</tr>
<tr>
<td>Russo and Bressler [1981]</td>
<td>Hamra Red Mediterranean soil</td>
<td>0.4–1.1</td>
<td>14–39 H</td>
<td>100</td>
</tr>
<tr>
<td>Luxmoore et al. [1981]</td>
<td>weathered shale subsoil</td>
<td>0.8</td>
<td>&lt;2 H</td>
<td>14</td>
</tr>
<tr>
<td>Sisson and Wierenga [1981]</td>
<td>silty clay loam soil (alluvial)</td>
<td>0.6</td>
<td>0.1 H</td>
<td>6</td>
</tr>
<tr>
<td>Viera et al. [1981]</td>
<td>Yolo soil (alluvial fan)</td>
<td>0.9</td>
<td>15 H</td>
<td>100</td>
</tr>
<tr>
<td>Devary and Doctor [1982]</td>
<td>alluvial aquifer (flood gravels)</td>
<td>0.8</td>
<td>820 H</td>
<td>5,000</td>
</tr>
<tr>
<td>Byers and Stephens [1983]</td>
<td>fluvial sand</td>
<td>0.9</td>
<td>0.1 V</td>
<td>5</td>
</tr>
<tr>
<td>Hoeksema and Kitaniidis [1985]</td>
<td>sandstone aquifer</td>
<td>0.6</td>
<td>45,000 H</td>
<td>$5 \times 10^5$</td>
</tr>
<tr>
<td>Hufschmied [1985]</td>
<td>sand and gravel</td>
<td>1.9</td>
<td>0.5 V</td>
<td>20</td>
</tr>
<tr>
<td>Sudicky [1985]</td>
<td>outwash sand</td>
<td>0.6</td>
<td>0.1 V</td>
<td>20</td>
</tr>
</tbody>
</table>

Correlation scales based on $e^{-1}$ correlation distance; H, horizontal sampling, V, vertical sampling.

Graham and Neff (1994) Floridan Aquifer (limestone)
1.26 10,000m H 2x10^5 m
Fig. 8. Hypothetical $\ln K$ variogram illustrating the notion of scale-dependent correlation scales.
Basic Problems Addressed

• Estimating the spatial distribution of soil properties/aquifer parameters given scattered measurements of these parameters or some process dependent on these parameters (geostatistics)

• Quantify the effect of estimation error in model predictions when spatial distributions of parameters, ICs and BCs are uncertain (stochastic modeling)

• Incorporating measurements into models using physically derived probabilistic relationships to improve predictions (data assimilation)
Stochastic Approaches

• Empirical- model uncertainty based on history of past behavior of available spatial or temporal measurements (times series analysis, kriging)

• Theoretical- derive model uncertainty from physically based equations

• Some combination of above
Geostatistical Analysis

• Provides a set of statistical tools for incorporating space-time coordinates of natural resources observations in data processing
• Describes patterns of spatial dependence of an attribute of interest and relates them to distributions of sources of these patterns
• Builds a probabilistic model of the spatial distribution of the attribute and its sources
• Estimates the spatial distribution at unmeasured locations
• Predicts accuracy of the spatial distribution
• Uses the estimated distribution to make predictions of status, value, risk etc.
Stochastic Modeling

• Combines physics of flow and transport determined in the lab with uncertainty and natural variability of hydrologic properties, model parameters and model predictions common in field scale systems

• Tries to account for variability on flow and transport predictions without making measurement task impossible

• We will specifically look at
  – Darcy’s equation
  – 3-D Saturated flow equation
  – 2-D aquifer equation
  – Richards (unsaturated flow) equation
  – Solute transport (advection-dispersion equation)
Data Assimilation

- Methodology to combine uncertain model predictions with error-prone measurement data from multiple sources to produce “optimal” predictions of the state and parameters of the system
- We will specifically look at
  - Kriging and its variants (characterization of a time invariant system using data derived or model derived first and second order moments)
  - Kalman Filtering (characterization of time-dependent system at most recent measurement time using model derived approximate first and second order moments)
  - Ensemble Kalman Filtering (characterization of time dependent system using an ensemble of models to estimate first and second order moments)
  - Particle Filtering (characterization of time dependent system using an ensemble of models to estimate entire pdf of system)
  - Generalized Likelihood Uncertainty estimation (forecasting of a time dependent system using an ensemble of models with parameter combinations that show similar performance in reproducing observational behavior)
Goal of this course

- Conquer jargon barrier so that can read stochastic literature with some degree of comfort
- Introduce basic tools so can evaluate whether tools can/should be incorporated into your own research